

On the maximum drawdown during speculative bubbles

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Abstract

A taxonomy of large financial crashes proposed in the literature locates the burst of speculative bubbles due to endogenous causes in the framework of extreme stock market crashes, defined as falls of market prices that are outlier with respect to the bulk of drawdown price movement distribution. This paper goes on deeper in the analysis providing a further characterization of the rising part of such selected bubbles through the examination of drawdown and maximum drawdown movement of indices prices. The analysis of drawdown duration is also performed and it is the core of the risk measure estimated here.

Key words Risk measure, drawdown, speculative bubbles.

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89.90.+n Other areas of general interest to physicist

1 Introduction

An insight in the long term behavior of portfolios is a delicate task in long term investment strategies. The need to consider extreme financial market events encompasses the investigation of large financial crashes, that were already classified as outliers with respect to the bulk of market drops, and often associated to the burst of speculative bubbles due to endogenous causes [1].

This paper aims at extracting risk features that characterize the rising part of such speculative bubbles.

Our data selection relies on the huge data analyses worked out by Johansen and Sornette [1, 2]. In their several papers, they develop and support through empirical evidence the theory describing speculative bubbles due to endogenous causes like as systems close to some rupture point. In particular large market

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indices drops ending speculative bubbles due to endogenous causes have been characterized through the occurrence of log-periodic-power laws (LPPL), interpreted as a signature of underlying cooperative phenomena among market agents (Fig. 1, Tab. 1). These results were related to analyses of drawdown movements of market indices prices (DD), thus providing systematic taxonomy of crashes [1, 3, 4]. Our analysis starts from this point and goes on inquiring drawdown market prices movements duration and size. The analysis of duration of drawdown movement is relevant in itself, considering the key role of time in catastrophic events like as large financial crashes are, that show a drop of prices within short time intervals [1, 2, 5]. The analysis of drawdown size requires as a preliminary step the selection among the several different definitions of drawdown that were proposed in literature. Results depend strictly on the measure chosen for drawdown, and on the actual data set [1, 5, 6, 7]. A first analysis classified large stock markets indices crashes as outliers whether considering the stretched exponential distribution of market drops [6]. A further analysis for noise filtered drawdowns allowed to calibrate the parameter in order to avoid outliers [5]. Other studies using different definitions of drawdowns and considering different data sets proposed for returns drawdown probability density fitting stable models [8], the generalized Pareto distribution (GPD) [7, 8], and the modified GPD. The worst scenario of maximum drawdown movements (MDD) has also been examined. Results depend also on the time window on which the maximum is estimated. For example in [7] the time window selection corresponds approximately to half an year, the entire data sets cover more than 20 years, and the results are used for probability distribution calibration. In [8] moving windows of 63 data each are used on datasets including at least 15 years. In both cases the long time series of market indices data that were used include lower and bigger financial crashes, the rise and the deflate of bubbles, till to the return to fundamentals, and the several changes of regimes included in societal and trading dynamics spanning the last 15-20 years. Therefore, the maximum drawdown measure calculated on moving windows gives samples of a process obeying several different dynamics. Here we aim at using a different approach for drawdown modeling. Instead of splitting the entire time series that are available, we extrapolate the probability of drawdown and maximum drawdown during the rising part of the speculative bubble, looking for the extraction of common features.

We compare the results obtained on the DD through the estimate of risk measures. Risk exposure in financial markets has been described through several measures (VaR, CVaR, etc.) based on statistical properties of data that do not consider the order of data sequences. In fact empirical mean, variance, as well as higher order moments of probability distributions are invariant under data shuffle. But long downward trends containing long lasting sequences of consecutive drawdown price movements could suggest investors to withdraw from the market, and they can quite force small investors to such a choice [9]. Therefore, measures of risk based both on the duration of consecutive market drops and on the maximum drawdown can play a relevant role in driving investment strategies.

We stress again that we use here a methodological approach deeply different from other comparative studies on risk measures, because we aim at extracting features of periods (the rising part of speculative bubbles due to endogenous causes), that are supposed to be driven by the same kind of dynamics, and we proceed comparing them with the entire data set available, whilst other studies that compare risk measures on long time windows do not take into account the evolution of dynamics driving stock markets across decades [8].

2 Large stock market indices crashes

A series of papers about speculative bubbles proposes an explanation of large financial crashes due to causes endogenous to the market. Similarities with critical phenomena like earthquakes and the sound emission in materials close to the rupture point led the research to the detection of cooperative underlying phenomena evidenced through discrete scale invariance in financial data (FX, Gold, stock market indices) close to large crashes that can thus be well described as critical points [2]. In the case of market indices the logarithm of index values close to the crash time is described by a characteristic LPPL:

$$F(t) \simeq A + B(t_c - t)^m + C(t_c - t)^m \cos(\omega \ln(t_c - t) - \phi) \quad t < t_c \quad (1)$$

where t_c is the crash time, and A, B, C, m, ω, ϕ , and t_c are parameters to be estimated via numerical optimization. A further taxonomy that distinguishes the causes that generate crashes can be found in [1] and confirms the detection of LPPL as a hallmark of speculative bubbles due to causes endogenous to the market [1, 10, 11] cross-validated with the amplitude of the crash ending the bubble as an outlier from probability distribution of the bulk of price variations [4, 6]. Stock market indices datasets reported in Tab. 1 were selected lying on the results of [1] in which the selection of an outlier market drop is used in itself as a definition of crash and then it is cross-validated with the occurrence of LPPL. The rising period of the bubble in which the LPPLs occur is selected starting from a lowest point before the rise of the bubble and ending at the maximum value of the index [2].

3 Drawdown duration distribution

The time plays a key role in crash definition [1, 2] and in the reaction of investors to the change of market condition, but most attention up to now was paid to the MDD than to the time in which it occurs, although the time measure was proposed as a first step towards the definition of some “drawdown velocity” [1]. Therefore, for each selected data set we exploit the duration of drawdowns, defined in the most simple way as the number of time steps from a local peak to the next local minimum, corresponding to the so called “pure” drawdowns of [1, 2, 5].

A first empirical result for drawdown duration is shown in [7], where the negative binomial or the Poisson distribution is proposed for modeling drawdown duration, and the Gamma or Pareto distribution for modeling the maximum drawdown.

Let $\{p_t\}_{t=1,T}$ be a time series of a stock market index daily closure price, and let $r_t = \log(p_t/p_{t-1})$ be the usual log-returns. The following definitions are in accord with the drawdown definition used in [7], apart from a normalization factor that is not relevant for our analysis (Fig. 2).

Definition 1 Let p_k be a local maximum, and p_l be the next local minimum:

$$p_k > p_{k+1} > \dots > p_l, \text{ with } l - k \geq 1, k \geq 1, \text{ and } p_k > p_{k-1}, p_{l+1} > p_l. \quad (2)$$

A pure DD (size) is defined as the partial sum of daily returns

$$DD_{k,l} = |r_{k+1} + r_{k+2} + \dots + r_l| = \left| \sum_{j=1}^{l-k} r_{k+j} \right| = \left| \log \frac{p_l}{p_k} \right| \quad (3)$$

Definition 2 The drawdown duration is the time length $l - k$ of the sequence of negative returns defined through (2).

DD duration was estimated on each selected time series. The power law was fitted to DD duration histograms. On the sets of the rising parts of selected bubbles shown in Fig. 1 the power law exponent ranges from 0.46 to 1.7, having a mean value $\alpha \simeq 1$. On the entire data sets α ranges from 0.67 to 1.10, with α mean value $\alpha \simeq 0.94$, accordingly to the presence of longer and deeper crashes in the enlarged time domain. The Poisson distribution gives a poor fit of DD duration histogram on the lowest duration values, and no useful result comes from the negative binomial fit. Therefore, we are going to use the empirical estimates for the risk measure computation (Fig. 3).

Studies on speculative bubbles have attempted to extract common features beyond LPPL. A first attempt [7] to relate GARCH parameters with the prediction of the volatility connected to the drawdown movement is made clearer in [12]: the presence of a bubble drives the high volatility in the GARCH model, but the reversal implication does not hold. The power law exponents of drawdown duration decay seem to evidence some general feature of drawdown duration interesting in itself.

The grouping of the exponents in a range supports the hypothesis of close risk profiles across international markets, from the DD duration point of view. However, a further correlation analysis comparing the α estimated on each series and the LPPL parameters ω and z does not show significative empirical correlation, like it was stated for GARCH parameters.

4 Relationships with other drawdown definitions

The delicate task of defining what is a crash, a large drop or a significant change of regime is far from being well assessed [12]. Also the closely connected definitions of drawdown market movements analyzed in the literature are not homogeneous [1, 9, 13]. Therefore we are going to examine the relationship between

the so called *pure drawdown* (DD), *ϵ -drawdown* (ϵ DD) [1, 7], that are used in the theory of speculative bubbles, and the maximum drawdown (\bar{D}) used in [9, 13], that provides a risk measure interestingly related to the Calmar and Sharpe ratio.

Definition 3 The maximum drawdown M is the longest partial sum of daily returns

$$M = \max\{DD_{k,l} \mid \text{conditions (2) hold}\} \quad (4)$$

The definition of ϵ DD relaxes the condition (2) allowing for small rises of values during the decreasing sequence. This description can be formalized through the following set of conditions

$$k = \arg(\max_s(s-l)), \text{ s.t. } p_k > p_{k+1} > p_{k+2} - \epsilon > \dots > p_l - \epsilon, l - k \geq 1, k \geq 1, \\ p_k > p_{k-1}, p_k > p_i, i = 1, \dots, l, p_{l+1} > p_l + \epsilon, \epsilon \geq 0. \quad (5)$$

and therefore to the following

Definition 4 The ϵ DD ending at time l is given by

$$\epsilon DD_l = \{DD_l \mid \text{conditions (5) hold}\}. \quad (6)$$

This definition states that price growth below a certain magnitude ϵ is ignored, and it serves to filter noise. If $\epsilon = 0$ the definition corresponds to “pure” drawdown; filter size was chosen as a function of the empirical volatility σ ($\epsilon = 0, \sigma/4, \sigma/2$) [1]. This definition includes in the particular case $\epsilon = 0$ the definition of pure drawdown ending at time t (DD_t).

Definition 5 The duration of ϵDD_l is given by

$$\max_s\{(s-l) \mid \text{conditions (5) hold}\} \quad (7)$$

\bar{D} will be the worst loss, i.e. max of the above.

The definition used in [13] starts from a stochastic processes approach:

Definition 6 Let $\{X(t)\}$ a stochastic process. \bar{D} is given by

$$\bar{D} = \sup_{t \in [0, T]} (\sup_{s \in [0, t]} X(s) - X(t)). \quad (8)$$

This measure gives the range from the maximum to the minimum anytime the maximum precedes the minimum. \bar{D} was calculated using $X(t) = \log(p_t)$, so to deal with returns.

Although the definition of ϵ DD was aimed only at filtering the noise the following remark holds:

Remark 1 Let $X(s) = \log(p_s)$. For each t ending time of an ϵDD_t :

$$\epsilon DD_t < \bar{D}$$

Remark 2 \bar{D} is an ϵDD_t for $X(t) = \log(p_t)$, $\epsilon > \max_t |p_t - p_{t-1}|$.

Henceforth risk measures based on DD and \bar{D} will provide bounds to ϵ DD. The following chain of inequalities is a direct consequence of the weakening of

conditions in the definitions.

Lemma 1 In accord with the definitions reported above:

$$\begin{aligned} DD_t &< \epsilon DD_t < \bar{D} \\ M &< \bar{D} \end{aligned} \tag{9}$$

The size of drawdowns described by \bar{D} is higher of M , due to the weakening of the descent conditions, and provides a worse scenario.

The same remark allow to conclude that the same inequality holds on the duration of DD_t and ϵDD_t .

5 A measure of risk based on drawdown movements size and duration

Stock market indices actually are a particular portfolio, basing on a weighted mean of selected stock prices, and to buy/sell stock market indices has the meaning to buy/sell a previous selected financial product replica of the index (Exchange Traded Funds, certificates). Portfolio risk measures considering DD and MDD should be used in a complementary way with respect to the traditional ones (VaR, ES) at least in the case of Stable Paretian distribution [8]. This section aims at comparing the behavior of a risk measure based on drawdown movement size on the rising part of speculative bubbles and on the entire time series. Results of [1] that classify as outliers with respect to the stretched exponential distribution the large financial crashes rising from the burst of speculative bubbles due to endogenous causes were extended considering coarse-grained drawdowns (ϵ -drawdowns) [5], and later the GPD distribution for negative tails was tested [7, 8, 14, 15]. These last results go beyond the mere distribution hypothesis testing, relying on literature on extreme events, that proposes the GPD, as a universal description of the tail of distributions of Peaks-Over-Thresholds. The same approach can be used for the MDD estimates. Although the duration plays a key role in drawdowns little or no analysis has been reported on the literature on drawdown duration [7]. Therefore, we perform analyses on the joint probability of both drawdown size and drawdown duration. The following measure of drawdowns

$$Pr\{DD < s\} = \sum_{d=1}^{\infty} Pr\{DD_{k,l} < s \mid l - k = d\} Pr\{l - k = d\} \tag{10}$$

is estimated on the rising part of each selected bubble and on the entire time series of the corresponding stock market index. This approach differs from the one used in [7, 14], that relies on probability obtained as a best fit of empirical drawdown size distribution, but that is based on independence hypothesis for drawdown duration modeling.

The weight of the biggest crashes in the entire time series evidences in lowering curves corresponding to entire time series.

Following the approach of [7] we fit the GPD

$$G_{\xi,\beta}(x) = 1 - (1 + \xi * x/\beta)^{-\frac{1}{\xi}}, \quad x \in D(\xi, \beta) \quad (11)$$

on the set of (10) estimated on the rising part and on the set of entire time series (Figs 4, 5). This result provides the mean behavior of drawdown to which refer in case of speculative bubbles, and evidences the difference with respect to the common behavior of the entire time series. On the rising part of the bubble $\beta = 0.0089(0.0085, 0.0093)$, $\xi = 0.4273(0.3783, 0.4763)$; on the entire time series $\beta = 0.0153(0.0148, 0.0158)$, $\xi = 0.3012(0.2698, 0.3326)$. Values reported in parenthesis are 95% confidence interval. The ranges of ξ and β parameters are not overlapping. The change of parameters value gives also another measure of large drops considered in the entire time series if compared with the drops experienced during the bubble. For each selected bubble Fig. 5 shows the cumulative function (10) of both the rising part of the bubble and the entire index. The closest the functions, the closest the drawdown structures are, and the lowest the impact of the outlier, that implies also that minor crashes in the rising part of the bubble have a relevant weight. As an example, drops during the rising part of the speculative bubble of Nasdaq collapsed in April 2000 had an intensity close to the Nasdaq crashes of the previous years, leading to highly overlapping curves.

Referring to Fig. 4, “fit 2” is the regression on the sets of (10) estimated on the rising part of speculative bubbles. “fit 2” separates DD behavior on the entire data set from 1/3 of (10) estimated on the rising part of speculative bubbles, corresponding to series exhibiting smaller and/or shorter DD.

Risk profiles of rising part/entire time series are not completely separated, but the result allow to use lower risk profiles in the rising part of speculative bubbles whether carefully encapsulated in a predictive scheme of burst of bubbles like [12].

6 Maximum of drawdown movements

Other measures of drawdown behavior were considered, in the case of the occurrence of the worst scenario. A discussion of coherence properties of measures DD-based is carried on [7, 9]. $Pr\{M < s\}$ was calculated in [7] conditioning to the duration, like in (10). The probability distribution of M was extracted through a segmentation of 15-20 years long time series. This procedure relies on the (undeclared) assumption that data in time windows considered obey a homogeneous underlying process, that of course is a very raw assumption. The need to split windows is due to the fact that only one M is available for each time series, but several samplings are needed in order to build a distribution. Here we follow a deeply different methodological approach, aiming at extracting features that can be shared by the rising part of speculative bubbles. Therefore we consider $max(DD_{k,l})$ conditioned to the duration and we estimate and

compare

$$E[M] = \sum_{d=1}^{\infty} \max\{DD_{k,l} \mid l - k = d\} Pr\{l - k = d\} \quad (12)$$

on each time series. Results are shown in Fig. 6. The leftmost picture reports the results on the rising part of the bubble, the rightmost on the entire time series. This estimate compares the size of maximum sequences of drops inside the bubble with the ones outside it, allowing to understand the impact of a huge financial crash with respect to the smaller crashes internal to the bubble. $E[M] \in (0.01, 0.08)$ (rising part) and $E[M] \in (0.05, 0.13)$ (entire series). Ranges are partially overlapping, remarking that some crashes classified as huge in some indices had a magnitude not so impressive for other indices.

Referring to Fig. 6, the discriminant analysis locates $E[M] \in (0.01, 0.05)$ for the rising part of speculative bubbles and $E[M] \in (0.08, 0.13)$ for the entire time series. Intermediate values $(0.05, 0.08)$ report the situation for the crashes of Nasdaq 100 in 1998 and 2000, DAX 40 in 1998, Argentina in 1997, Hong Kong in 1994 and 1997 (rising parts of the bubbles; on Fig. 1 labeled, respectively, Nasdaq 100_1998, Nasdaq 100_2000, DAX 40_1998, Arg Merval_1997, Hang Seng_1994 and Hang Seng_1997), and for FTSE 100, Chile General (Igap), Jakarta SE Composite (entire time series).

Fig. 7 shows $\bar{D} \in (0.04, 0.17)$ for the rising part of speculative bubbles; $\bar{D} \in (0.08, 0.35)$ for the entire time series.

The weakening of the descent conditions can also be addressed for the higher overlap in windows values. Anyway this measure is worth of being considered because it gives the very worst loss case in case of long time buy-and-hold strategies.

7 Conclusions and further developments

This paper focuses on risk measures based on drawdowns and has introduced the empirical estimate of duration in the loss function underlying a risk measure. The analysis of the duration of drawdown is relevant for fund managers, that can bear high volatility periods, but that risk to lose clients whether a large sequence of drops happens. Empirical results are used to extract characteristics common to the rising part of speculative bubbles and to evidence similarities and differences with respect to the entire time series. A comparison between risk measure DD-based shows the improvement in the discriminant analysis obtained by the introduction of DD duration. The approach used here is deeply far from the most common usage to split long time series or to simulate them basing on the raw hypothesis of a homogeneous process underlying long time series. Recently an alert system [12] was proposed in order to open the way to practical usage of the LPPL bubble theory for the forecast of the bubble ending time. We look forward for the embedding the present analysis into the predictive scheme of [12] for the calibration of investment strategies during the rising part of bubbles, before the expected crash time.

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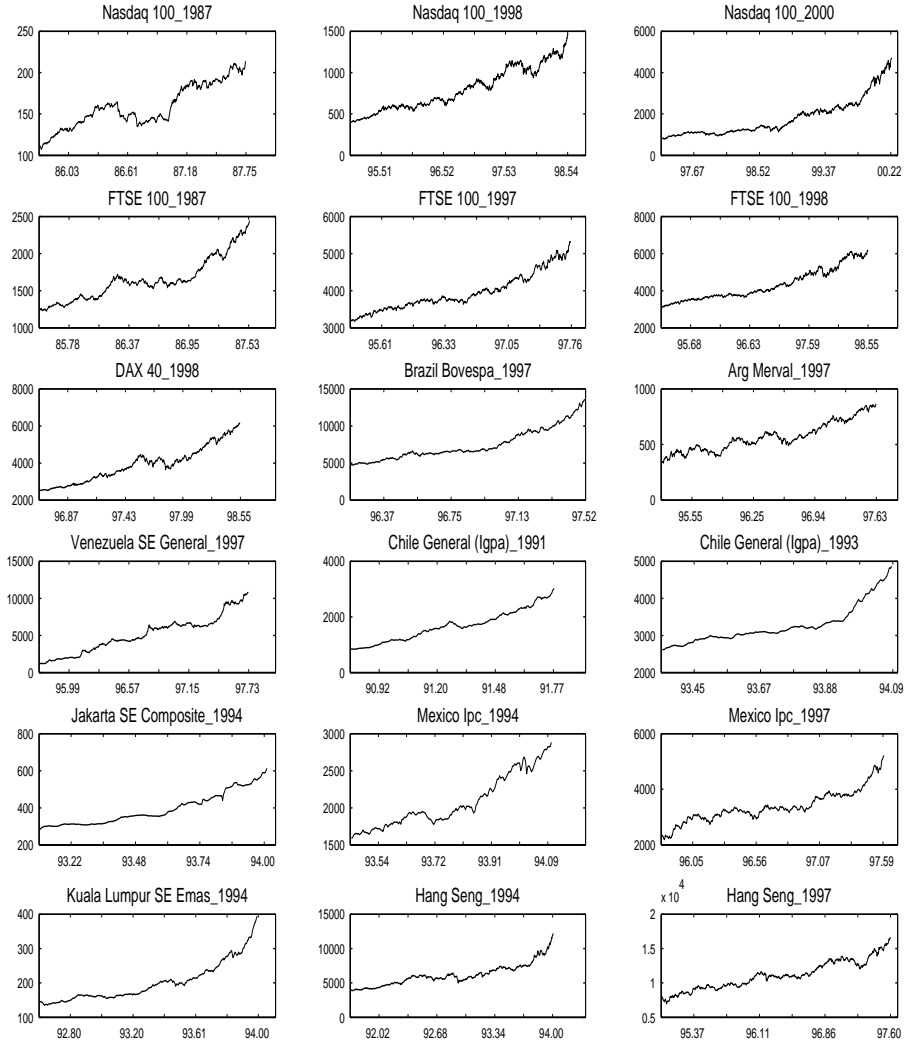


Figure 1: Rising part of speculative bubbles selected. Large financial crashes were chosen in accord to [1]. The complete list is given in Tab. 1. Time is reported on x axis, in accord with the notation used by [1]. y axis reports the value of each index.

Table 1: List of crashes. Data sets of rising part of speculative bubbles are chosen since the rise of the bubble to the expected crash time [2]

Index	Crash year	Rising part	Entire dataset
Nasdaq 100	1987	10/01/1985 - 10/02/1987	10/01/1987 - 12/31/2001
Nasdaq 100	1998	01/02/1995 - 07/20/1998	10/01/1987 - 12/31/2001
Nasdaq 100	2000	04/01/1997 - 03/27/2000	10/01/1987 - 12/31/2001
FTSE 100	1987	07/01/1985 - 07/16/1987	07/01/1985 - 12/31/2001
FTSE 100	1997	04/03/1995 - 10/03/1997	07/01/1985 - 12/31/2001
FTSE 100	1998	03/14/1995 - 07/20/1998	07/01/1985 - 12/31/2001
DAX 40	1998	08/06/1996 - 07/20/1998	08/06/1996 - 12/31/2001
Brazil Bovespa	1997	03/06/1996 - 07/08/1997	03/06/1996 - 12/29/2000
Arg Merval	1997	03/15/1995 - 08/20/1997	03/15/1995 - 12/29/2000
Venezuela SE Gen	1997	09/12/1995 - 09/23/1997	09/12/1995 - 12/29/2000
Chile General (Igpa)	1991	10/10/1990 - 10/08/1991	10/10/1990 - 12/31/1996
Chile General (Igpa)	1993	05/10/1993 - 02/04/1994	10/10/1990 - 12/31/1996
Mexico Ipc	1994	06/15/1993 - 02/08/1994	06/15/1993 - 12/29/2000
Mexico Ipc	1997	10/18/1995 - 08/06/1997	06/15/1993 - 12/29/2000
Kuala Lumpur SE Emas	1994	08/06/1992 - 01/04/1994	08/06/1992 - 21/31/1996
Hang Seng	1994	09/12/1991 - 01/04/1994	09/12/1991 - 12/29/2000
Hang Seng	1997	01/02/1995 - 08/07/1997	09/12/1991 - 12/29/2000

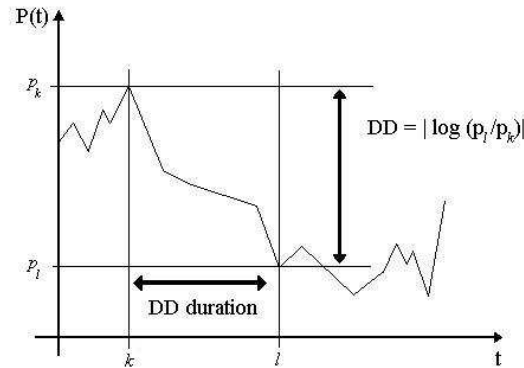


Figure 2: Pure DD definition in accord to (3)

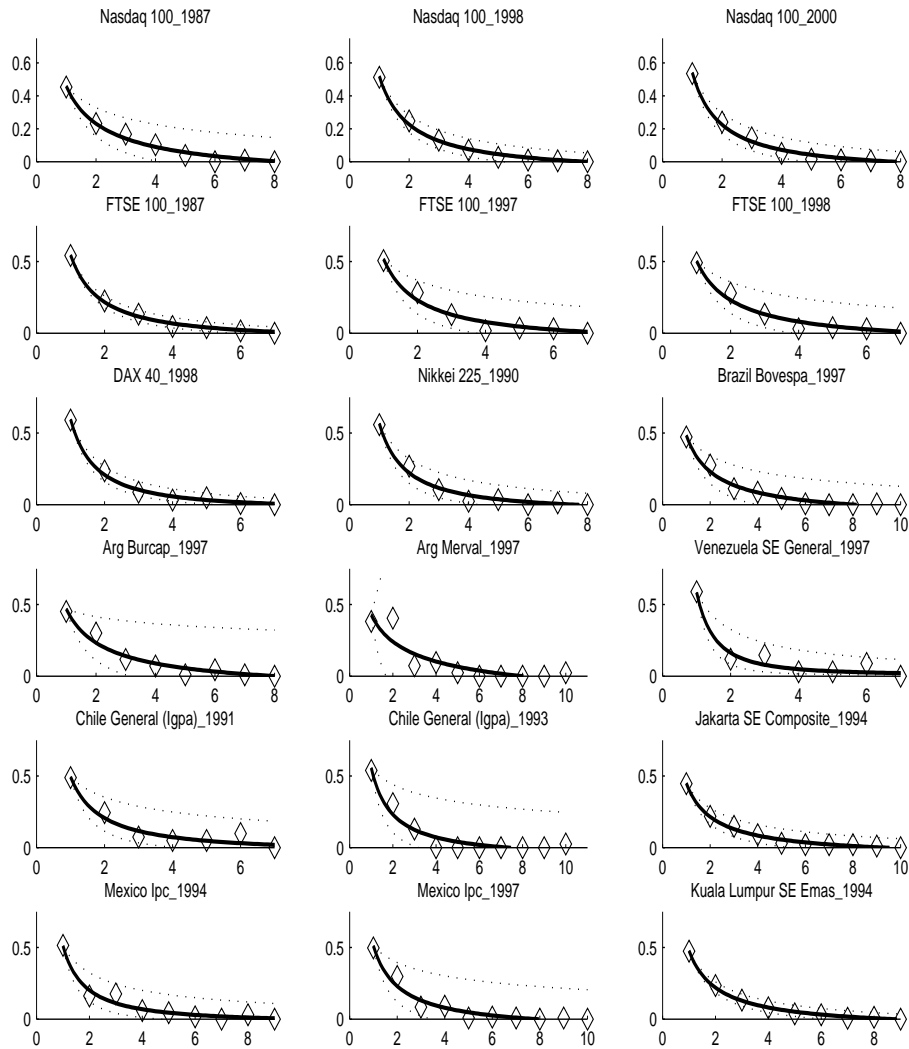


Figure 3: Histogram of the duration - rising parts of the speculative bubbles. Usual histogram bars were substituted by diamonds because of graphic clarity. x -axes report the DD duration in term of days; y -axes report their frequencies. On each example the solid line is the power law regression curve; the dotted line is drawn using the bounds of the 95% confidence intervals. The decay exponents $\alpha \in [0.46, 1.7]$ where 0.46 is the minimum value and 1.7 is the maximum value over all the reported samples; the mean value of α is approximately at 1.

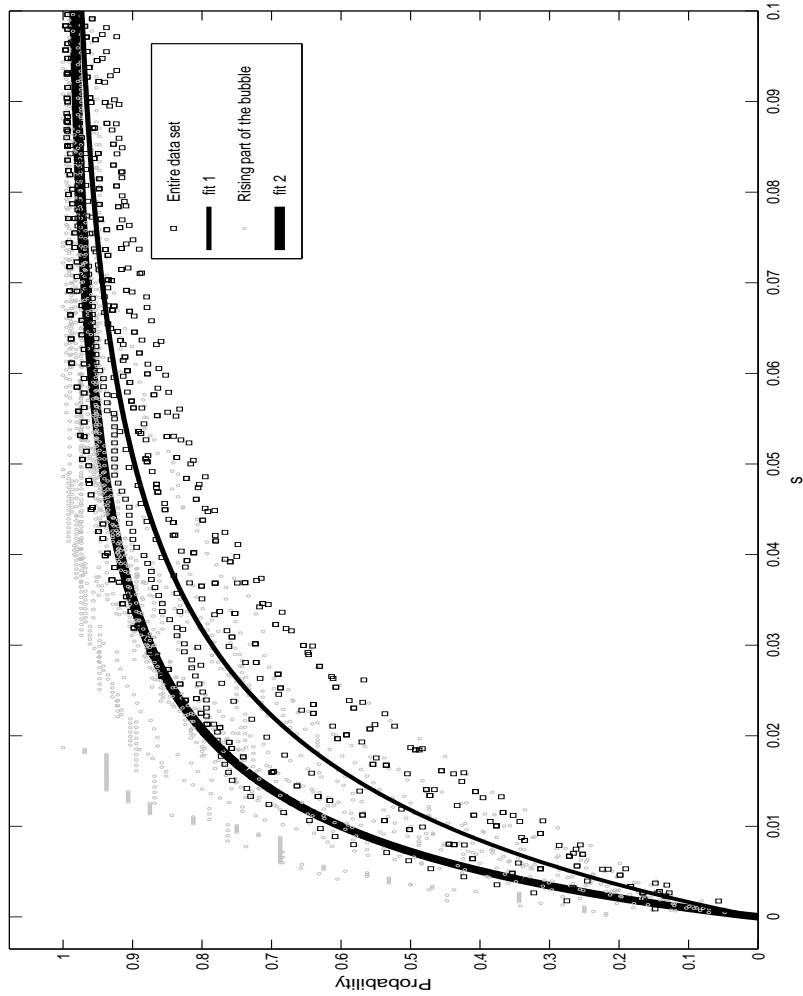


Figure 4: Best fit of empirical estimates of (10) through (11). x -axis reports the values of s ; y -axis the cumulative probability. The curve labeled “fit1” is the regression curve obtained by fitting (11) on the empirical estimates of (10) on the set of entire time series. Parameters values are $\beta = 0.0153(0.0148, 0.0158)$, $\xi = 0.3012(0.2698, 0.3326)$. The curve labeled “fit2” is the regression curve obtained by fitting (11) on the empirical estimates of (10) on the set of rising part of bubbles. Parameters values are $\beta = 0.0089(0.0085, 0.0093)$, $\xi = 0.4273(0.3783, 0.4763)$. Values reported in parentheses are 95% confidence interval.

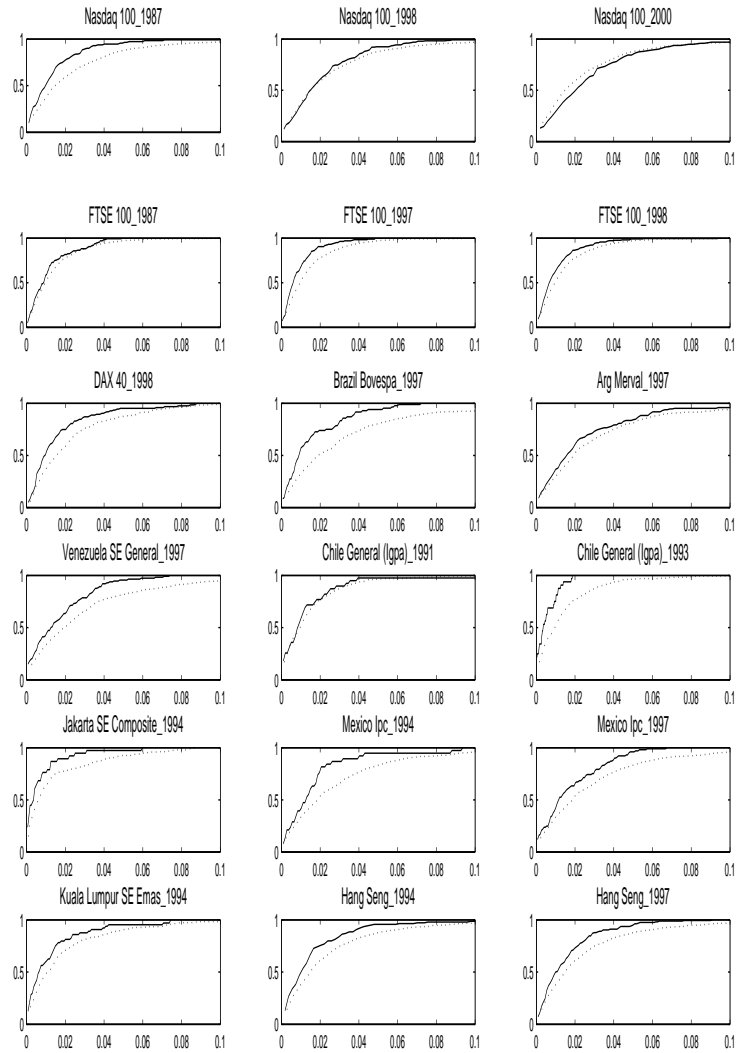


Figure 5: Empirical estimates of (10). x -axes reports the value of s ; y -axes the cumulative probability. Each figure reports (10) estimated on the rising part of a selected bubble (solid lines) and compared with (10) estimated on the entire time series (dashed lines).

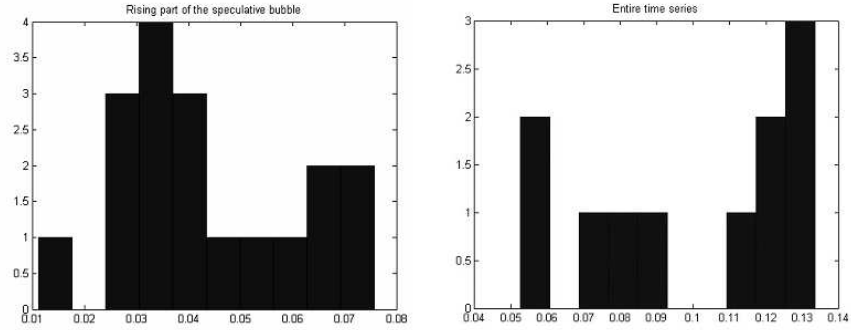


Figure 6: Histogram of maxima of pure drawdown (12). x -axes reports values of $E[M]$; y -axes reports the counting of them. On the set of rising part of speculative bubbles the range is $(0.01, 0.08)$; on the set of entire time series the range is $(0.05, 0.13)$.

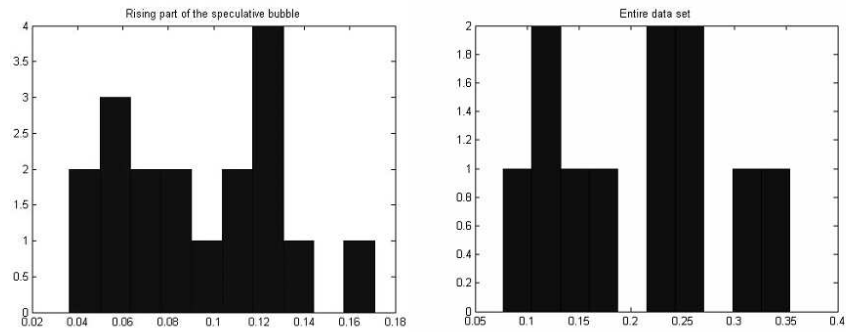


Figure 7: Histogram of \bar{D} (8). x -axes reports values of \bar{D} ; y -axes reports the counting of them. On the set of rising part of speculative bubbles the range is $(0.04, 0.17)$; on the set of entire time series the range is $(0.08, 0.35)$.