

Portfolio Construction with Asymmetric Risk Measures

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Abstract

Portfolio construction techniques seek an optimal trade-off between the portfolio's mean return and the associated risk. Given that risk may not be properly described by return volatility we examine alternative measures that account for the asymmetric nature of risk, i.e. downside risk. In particular, we optimize portfolios with respect to semideviation, semivariance, skewness, loss-oriented utility or maximum drawdown in an empirical out-of-sample setting. To avoid bias due to misspecified return forecasts we assume perfect foresight of returns whereas alternative risk measures are being estimated from historical data. Our empirical results indicate that downside portfolio risk is reduced for most of the investigated measures. We find significant risk reductions for semivariance, semideviation, maximum drawdown and loss-oriented utility. However, lacking persistence both tested skewness measures are rendered rather useless for portfolio construction purposes.

Keywords: portfolio optimization, asymmetric risk measure, skewness, maximum drawdown, semideviation, semivariance, lower partial moments, utility function

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1 Introduction

Since Markowitz (1952) has lifted portfolio construction to a scientific level, a number of shortcomings of the classic mean-variance optimization approach have been discussed controversially in literature. These are in particular the static one period character, the high sensitivity of the optimization outcomes with respect to small changes in the inputs and the use of volatility as a risk measure. In this paper we focus on the latter. In fact, being a symmetric risk measure, volatility does not seem to fit most investors' definition of risk as the peril to end up with less than expected. Volatility counts both negative and positive surprises as risk, not distinguishing between cold and warm rain.

Tobin (1958) was one of the first to show that volatility can only be the risk measure of choice for quadratic utility functions or, otherwise, normally distributed returns. Both assumptions have been proven to be unsustainable, see e.g. Mandelbrot (1963), Fama (1965) for non-normality and Rubinstein (1973) or von Neumann and Morgenstern (1947) for properties of utility functions. To account for the asymmetric nature of risk Roy (1952) adds a criterion to Markowitz' efficient frontier which selects the efficient portfolio with the lowest probability to fall short of a given target return. Markowitz (1959) proposes a portfolio optimization procedure based on the semivariance measure. Fishburn (1977) employs a piecewise defined utility function model that translates downside risk depending on a risk aversion parameter and a target return. His findings indicate that investors actually show a significant change in their risk perception below this individual threshold. Hence, there is a broad consensus among both researchers and practitioners that asymmetry is a reasonable property for any risk measure, what disqualifies volatility. Nonetheless, although a number of alternatives have been proposed the mean-variance approach is still prevalent. However, none of them has proven to be appropriate to a broad society so far. Whatever the choice of risk measure, under perfect information an optimizer will certainly find risk-minimized portfolios for any given sample. In practice, however, portfolio construction takes place in an ex-ante context relying on forecasts of the respective risk measure that have to be derived from historical data. Therefore, forecastability of a risk measure is a prerequisite in practice. This is an important challenge because risk structures (think of correlations) are known to be unstable over time. For instance, the instability of beta factors and, as a consequence, covariance matrices and other correlation-based

risk measures has been documented by numerous studies.¹ Although it is important to consider alternative risk measures in an empirical out-of-sample setting this has not been addressed intensively in the literature. Bertsimas, Lauprete and Samarov (2004) apply a mean-shortfall optimization procedure to empirical data, but not by means of a backtest. Guidolin and Timmermann (2006) propose various econometric models to describe the term structure of value at risk and expected shortfall risk measures and compare them out-of-sample within a strategic asset allocation problem.

This paper adds to the existing evidence by empirically examining whether asymmetric risk measures are more useful to define risk in the context of portfolio construction. To do so, we substitute volatility as the objective function of portfolio optimization by a couple of alternative risk measures. We compare the out-of-sample performance characteristics of the mean variance based investment strategy to those of other risk philosophies in an empirical backtest.

The paper is structured as follows: Section 2 introduces the alternative risk measures considered. Section 3 covers the empirical methodology. Section 4 discusses our empirical findings. Section 5 concludes.

2 Candidate Asymmetric Measures of Risk

All of the risk measures discussed in this section are downside risk oriented. In contrast to symmetric risk measures they only treat negative deviations as risk but not positive deviations.

Value-at-Risk

Given the random return R of a portfolio for a certain holding period the value at risk (VaR) determines the loss corresponding to the lower quantile of its distribution for a given (small) probability. Let the probability be $p = 0.01$, then

$$\text{VaR}_p(R) = -F_R^{-1}(0.01)W,$$

where F_R is the cumulative distribution function of R and W is the investor's initial wealth, a constant we neglect in the following. Thus, the VaR is neatly interpretable, providing a loss value that is not breached with a certain (high) probability. However, VaR ignores

¹See, among others, Fabozzi and Francis (1978), Bos and Newbold (1984) and Schwert and Seguin (1990) for the U.S., Wells (1994) for Sweden, Faff, Lee and Fry (1992) for Australia and finally Ebner and Neumann (2005) for Germany.

extreme events below the specified quantile. If optimization is not limited to certain distribution families, the optimizer will “sweep the dirt under the rug”, producing optimized portfolios with a low VaR but possibly fat tails beyond that threshold. Therefore, we think that VaR is better used for describing risk than for optimizing.

The drawback of VaR is mitigated by a related risk measure, the Conditional VaR (CVaR). The CVaR describes the expected loss for events within the small quantile determined by p . As presented by Bertsimas, Lauprete and Samarov (2004) CVaR exhibits convexity with respect to portfolio weights under fairly weak continuity assumptions. This facilitates mathematical optimization and contributes to the risk measure’s coherence (as defined by Artzner, Delbaen, Embrechts and Heath (1999)).

However, optimization for CVaR demands either an assumption about the return distribution or a considerable amount of observations below the target. Because both are typically not given in a portfolio context, we exclude CVaR as well.

Lower Partial Moments (LPM)

Instead of CVaR we consider the related semideviation risk measure. It exhibits a linear evaluation of below-target returns as well, but directly defines the mean as the target return. Semideviation belongs to the broader class of Lower Partial Moments (LPM) risk measures that only take downside events below a given target into account. Returns below this threshold are evaluated by a penalty function that increases polynomially with the distance from target return.

An LPM risk measure is determined by two parameters. One is the exponent or degree k and the other is the target return τ . Given the return distribution R of a portfolio, a lower partial moment is defined by

$$LPM_{\tau,k}(R) = \mathbb{E} \left((\tau - R)^k \mid R < \tau \right) \cdot P(R < \tau).$$

Common parameter choices are $k = 1$ or $k = 2$ and $\tau = \mathbb{E}(R)$, yielding the semideviation

$$SD(R) = \mathbb{E} \left((R - \mathbb{E}R) \mid R < \mathbb{E}R \right) \cdot P(R < \mathbb{E}R),$$

and the semivariance of R ,

$$SV(R) = \mathbb{E} \left((R - \mathbb{E}R)^2 \mid R < \mathbb{E}R \right) \cdot P(R < \mathbb{E}R).$$

Other typical values of τ are the risk-free rate or a zero return. These refer to the risk of falling behind opportunity costs or to the risk of realizing an absolute loss, respectively. The parameter k serves as a risk aversion parameter—losses are penalized with power k . For a detailed discussion of LPM properties see, for example, Harlow and Rao (1989) and Schmidt von Rhein (2002). Porter (1974) and Bawa (1975) show that the semivariance measure is consistent with the concept of stochastic dominance. An analogous proof for semideviation is given by Ogryczak and Ruszczyński (1998).

LPMs entail considerable computational difficulties, e.g. a portfolio’s LPM cannot be expressed as a function of security LPMs as it is the case for volatility. This contributes to the limited application of these risk measures in practice. For our study we take the computational burden and determine semideviation and semivariance. Let $(R_t)_{t=1,\dots,T}$, $R_t \in \mathbb{R}^N$ denote the sample of return (vector) realizations and let \mathbf{x} be the vector of portfolio weights. Then the vector of portfolio returns obtains as $(X_t)_{t=1,\dots,T}$ where $X_t = \mathbf{x}^T R_t$. Semideviation and semivariance of this portfolio are then computed as follows:

$$f(\mathbf{x}) = \text{SD}(X) = \frac{1}{T} \sum_{t=1}^T \min \{ (X_t - \bar{X}), 0 \}$$

$$f(\mathbf{x}) = \text{SV}(X) = \frac{1}{T} \sum_{t=1}^T \min \{ (X_t - \bar{X}), 0 \}^2$$

where \bar{X} denotes the mean of X estimated by $\bar{X} = \frac{1}{T} \sum_t X_t$.²

To summarize, we conduct two LPM optimizations,

$$f(\mathbf{x}) = \text{SD}(\mathbf{x}^T R) \rightarrow \min,$$

and

$$f(\mathbf{x}) = \text{SV}(\mathbf{x}^T R) \rightarrow \min.$$

The former describes the loss that is to be expected from an investment in the respective portfolio, the latter separates variance into upside and downside deviation and measures the second one.

A Loss-Oriented Utility Function

²Alternative ways of computing semideviation and semivariance have been proposed by Bawa and Lindenberg (1977) and Markowitz (1959) that save computational efforts at the cost of a certain inaccuracy. Given today’s computing power we neglect these procedures.

Mean-variance optimization is often put into a utility maximization framework. The investor is supposed to maximize the expected utility u of his portfolio return distribution, thereby solving

$$f(\mathbf{x}) = \mathbb{E}(u(\mathbf{x}^T R)) \rightarrow \max$$

for a given random vector of market returns R . The approach builds on the renowned work of von Neumann and Morgenstern (1947). For ease of deduction, the utility function is assumed to be quadratic (or fitting its quadratic approximation very well). At least in this case mean-variance optimization is consistent with utility maximization (see Tobin (1958)). Although utility functions cannot be specified precisely (see e.g. Rubinstein (1973)), there are a few widely accepted characteristics of investor behaviour. These are $u' > 0, u'' < 0, u''' > 0$ giving a so-called decreasingly (absolute) risk averse investor with utility function u . The properties of u allow definitions of utility functions that match our intuition of an investor with a downside risk attitude. We define a utility function representing constant elasticity of substitution given by

$$u(x) = -\exp(-x).$$

With this utility function negative deviations from the mean are punished more as compared to the benefits of positive deviations. However, we do not want to maximize utility but to minimize risk. Therefore we adjust the target function for the mean as follows:

$$f(\mathbf{x}) = \mathbb{E}(u(\mathbf{x}^T R - \mathbb{E}\mathbf{x}^T R)) \rightarrow \max$$

Figure 1 about here.

Figure 1 depicts the loss-oriented utility function to be maximized. In particular, the penalty for loss grows exponentially with its size. Again, the computation of investor utility is not as convenient as for volatility. For a given time series vector $(R_t)_{t=1, \dots, T}$ and a specific vector \mathbf{x} of portfolio weights we have to determine

$$X = (X_t)_{t=1, \dots, T}, \quad X_t = \mathbf{x}^T R_t$$

$$\mathbb{E}(u(X - \bar{X})) = \frac{1}{T} \sum_{t=1}^T -\exp(-X_t + \bar{X}).$$

We emphasize that we do not claim to know the investor’s utility function. We rather optimize a function having returns as arguments that fits our goal to mitigate downside risks and can coincidentally be interpreted as one possible utility function.

Skewness

The skewness of a return distribution R is defined as

$$\gamma(R) = \frac{\mathbb{E}((R - \mathbb{E}R)^3)}{\sigma(R)^{3/2}}.$$

It quantifies the deviation of R from symmetry. The more positive the skewness measure becomes, the heavier the upper tails compared to the lower tails of a distribution. Figure 2 shows how downside risk decreases with positive skewness. This is because the positively skewed (or right-skewed) distribution has less events at the far lower tail at the cost of more events slightly below mean, when compared to a symmetric distribution.

Figure 2 about here.

However, skewness does not add value on its own, we also need to limit volatility. Otherwise optimized portfolios might exhibit the desired relation between the tails, i.e. skewness, but nonetheless fatter downside tails due to an overall rise in volatility. This point is illustrated in Figure 3 by comparing two equally-skewed distributions with differing volatilities. Apparently, this effect also holds for left-skewed distributions.

For that reason, we add a non-linear constraint that limits volatility to a fixed level while testing for the performance effects of alternative skewness levels. We will particularly restrict the estimated portfolio variance $\hat{\sigma}^2$ to be smaller than that of the benchmark, $\sigma_{\hat{B}M}^2$.

Figure 3 and 4 about here.

Despite the self-evident effect of skewness on downside risk, the empirical evidence for a skewness premium in asset prices, as reported by Kraus and Litzenberger (1976) with their 3M CAPM, has been questioned repeatedly, see the empirical study of Post and Vliet (2003), and must be regarded as an open issue.

For optimization purposes, we consider two skewness estimators. First we use the standard

skewness estimator yielding the optimization problem

$$f(x) = \hat{\gamma}(x^T R) \rightarrow \max,$$

$$s.t. \quad \hat{\sigma}^2(x^T R) \leq \hat{\sigma}_{BM}^2,$$

where $\hat{\sigma}$ and $\hat{\gamma}$ stand for estimated volatility and skewness respectively, i.e.

$$\hat{\gamma}(X) = \frac{1}{T} \sum_{t=1}^T \left(\frac{X_t - \hat{\mu}}{\hat{\sigma}(X)} \right)^3.$$

The second skewness estimator is deemed to be more robust. It is repeatedly mentioned in literature, that the straightforward estimator is not only biased but also highly dependent on sample outliers (see Kim and White (2004) and Harvey and Siddique (2000)). A few other, more robust skewness estimators have been discussed. For our study we choose a robust skewness measure introduced by Bowley (1920):

$$S_B(X) = \frac{Q_3(X) + Q_1(X) - 2Q_2(X)}{Q_3(X) - Q_1(X)},$$

where $Q_i(X)$ is the i th quartile of the sample $X = (X_i)_{i=1, \dots, T}$. This estimator should be less sensitive to outliers than the traditional one, which is of particular relevance in portfolio construction given that return data exhibit fat tails. The optimization task we formulate is

$$f(x) = S_B(x^T R) \rightarrow \max,$$

$$s.t. \quad \hat{\sigma}^2(x^T R) \leq \hat{\sigma}_{BM}^2.$$

Maximum Drawdown

With the maximum drawdown measure (MDD) risk is defined as the maximum percentage loss an asset experienced from its top valuation to its bottom valuation within a given sample period. An important advantage of this risk measure is that it is independent from the underlying return distribution and its properties since it entirely relies on historical returns.

For any given time series of (discrete) returns for a single security S , $(R_t^{(S)})_{t=1, \dots, T}$, we can

easily compute the corresponding time series of the portfolio value, if it solely consists of security S . This value is given by

$$V_0^{(S)} = 1, \quad V_t^{(S)} = V_{t-1}^{(S)}(1 + R_t^{(S)}/100), \quad t = 1, \dots, T.^3$$

In order to find out what was the worst period to be invested in S we calculate the MDD for period $[0, T]$ recursively. Given the Maximum Drawdown for the period $[0, T - 1]$, $\text{MDD}_{[0, T-1]}(S)$, we obtain

$$\text{MDD}_{[0, T]}(S) = \min \left\{ \frac{V_T - V_{\hat{t}}}{V_{\hat{t}}}, \text{MDD}_{[0, T-1]}(S) \right\}, \quad (2.1)$$

where $V_{\hat{t}}$ is the maximum value of the security held within $[0, T - 1]$.

Setting $\text{MDD}_{[0, 0]}(S) := \infty$, we compute $\text{MDD}_{[0, T]}(S)$ by stepping through the sample and applying equation (2.1). For ease of presentation we neglect the index for the time interval in the following. Figure 5 illustrates the MDD of a portfolio that has been optimized with respect to the MDD and compares it to the MDD one would have suffered holding the benchmark.

Figure 5 about here.

The optimization task we formulate for MDD is

$$f(\mathbf{x}) = \text{MDD}(\mathbf{x}^T R) \rightarrow \min.$$

Some analytical analysis on the MDD has been done by Magdon-Ismael, Atiya, Pratap and Abu-Mostafa (2004). Hamelink and Hoesli (2004) provide an empirical study of real estate portfolios and show that actual portfolio weights (of real-estate portfolios) are closer to the MDD-optimized portfolio than to the mean-variance-optimized. Acar and James (1997) present an empirical analysis reporting poor predictability of MDD. Burghardt, Duncan and Liu (2003) point out the behavior of drawdowns as a function of track record length, volatility, mean return and others. Johanson and Sornette (2001) show that drawdowns follow an exponential distribution after excluding an exceptional probability mass of about 2 % that corresponds to the worst drawdowns ever occurred. Nouri (2006) presents an

³Typically, returns are deducted from values, so one can save the double computation.

analysis of expected drawdowns under constant and stochastic volatility.

3 Empirical Methodology

In our empirical study we investigate the characteristics of the portfolio construction approaches discussed above for a European equity portfolio to be managed against a benchmark.

We examine discrete weekly return data of the Dow Jones Euro Stoxx 50 universe from January 1993 to April 2006 exclusive of ten names that have missing data for parts of the period. For the remaining $N = 40$ stocks we adjust the index weights proportionally to ensure that they sum up to 100 % and calculate adjusted benchmark returns accordingly. For our empirical examination we adopt the following investment strategy. We optimize portfolios under the risk measures defined above in a relative benchmark oriented setting and require all optimization solutions to realize at least the benchmark return and to satisfy a certain tracking error limit. The objective is to minimize risk according to the respective risk definition.

We rely on an estimation period of two years (104 observations) to determine the risk measures under investigation and reallocate the portfolio every three months (13 weeks). We assume perfect foresight with respect to the return expectations but not with respect to expected risk measures. Doing so we isolate the portfolio construction problem from the return forecasting problem by assuming perfect return forecasts while the risk measures under investigation are subject to empirical estimation based on past data.

To summarize, we examine six portfolio strategies based on the six objective functions introduced in the last section, together with the traditional mean-variance approach using volatility. Using $X = \mathbf{x}^T R$ the optimization tasks are the following:

1. Volatility: $f(\mathbf{x}) = \sigma(X) = (\mathbf{x}^T C \mathbf{x})^{1/2} \rightarrow \min,$
2. Semivariance: $f(\mathbf{x}) = SV(X) = \mathbb{E}((X - \mathbb{E}X)^2 | X < \mathbb{E}X) \cdot P(X < \mathbb{E}X) \rightarrow \min,$
3. Semideviation: $f(\mathbf{x}) = SD(X) = \mathbb{E}(X - \mathbb{E}X | X < \mathbb{E}X) \cdot P(X < \mathbb{E}X) \rightarrow \min,$
4. Utility: $f(\mathbf{x}) = \mathbb{E}(u(X - \mathbb{E}X)) \rightarrow \max,$
5. Standard skewness: $f(\mathbf{x}) = \hat{\gamma}(X) = \frac{1}{T} \sum_{t=1}^T \left(\frac{X_t - \hat{\mu}}{\hat{\sigma}(X)} \right)^3 \rightarrow \min,$
6. Robust skewness: $f(\mathbf{x}) = S_B(X) = \frac{Q_3 + Q_1 - 2Q_2}{Q_3 - Q_1} \rightarrow \max,$
7. Maximum Drawdown: $f(\mathbf{x}) = \text{MDD}(X) \rightarrow \min.$

We include volatility in order to compare asymmetric risk measures also with Markowitz' traditional measure. To complete the formulation of our optimization tasks we add the following constraints:

First, we constrain the tracking error τ . For a given vector of benchmark portfolio weights \mathbf{x}_{BM} and (optimized) weights \mathbf{x} we restrict the tracking error by

$$\tau(\mathbf{x}) = \sqrt{(\mathbf{x} - \mathbf{x}_{BM})^T C (\mathbf{x} - \mathbf{x}_{BM})} \leq \tau_{\max}.$$

The $N \times N$ -matrix C is the sample covariance matrix of the random vector of portfolio returns.⁴ We set an annual tracking error limit of 5%.⁵ This leaves a fairly large discretion to the optimizer to determine the weights and allows to obtain a quite clear picture about the impact of the optimization process.

Second, we restrict the portfolio return to reach at least benchmark level, i.e.

$$\boldsymbol{\mu}^T \mathbf{x} \geq \boldsymbol{\mu}^T \mathbf{x}_{BM}.$$

Therefore the estimated mean return (and, in our case, under perfect foresight, a realized return) of the optimized portfolio is greater or equal to the benchmark return. Note that this constraint avoids optimization solutions to reduce risk at the cost of return. However, it sets no incentive for the optimizer to go for higher returns on the other hand.

Third, when skewness is the objective we have already argued that volatility should be limited as well. Thus, for optimization problems 5 and 6 we add the following constraint:

$$\sigma(\mathbf{x}) = \sqrt{\mathbf{x}^T C \mathbf{x}} \leq \sigma_{BM}.$$

Beside these quadratic constraints we demand additional linear (in)equalities to be met. These are

$$\sum_{i=1}^N x_i = 1 \quad \text{and} \quad x_i \geq 0 \quad \forall i,$$

known as full investment and nonnegativity constraints. In summary, these assumptions and restrictions imitate a portfolio manager who adjusts his portfolio every three months for a certain risk objective and therefore considers data from a revolving two-year-period.

⁴We apply a straightforward estimator: $\sigma_{ij} = (T - 1)^{-1} \sum_{i=1}^T (R_i - \mathbb{E}R_i)(R_j - \mathbb{E}R_j)$.

⁵We therefore apply the rule for log-returns as a rule of thumb for discrete returns: $\tau_{\text{annual}} = \tau_{\text{weekly}} \cdot \sqrt{52}$. Since we optimize using weekly return and volatility data, the implemented tracking error limit is $\tau_{\max} = 5\% \cdot \sqrt{52} = 0.69\%$.

Our methodology regarding the optimization process itself is rather pragmatic. We employ Matlab’s flexible *fmincon* function for all alternatives except volatility for which we use the *quadprog* function. *fmincon* offers a trust region method intended to tackle large scale problems and a line search algorithm for medium scale tasks. After adjusting for the number of iterations and error tolerance, the medium scale method performed satisfactorily. In other words, we do not spend much effort on streamlining the optimization itself but focus on setting the objectives and interpreting the outcome.

4 Empirical Results

The results of the examined investment strategies are shown in Table 1 in a condensed form.

Table 1 about here.

The first three columns of Table 1 show the optimization impact on the average value of the respective risk measures. They indicate that optimization decreases risk for most of the considered objective functions. The average values of volatility, semideviation, semi-variance, utility and Maximum Drawdown from optimized portfolios are far below the risk levels of the benchmark. The fourth column of Table 1 shows, how much of the potential percentage improvement in the risk measures under investigation is reached by the optimization procedure. Scaling to percentages makes the rows comparable. Again, the five examined risk measures show remarkable average improvement under optimization when compared to simply buying and holding the benchmark.

However, for the two skewness related risk measures the average improvement is close to zero and the deviation is in fact many times greater than the absolute value of improvement (or impairment, if robust skewness is concerned). This indicates that skewness is an unreliable property for the assets under scrutiny. Evidently, the sensitivity towards sample outliers does not account for the lacking reliability of standard skewness. This can be deducted from the paltry performance of the robust skewness measure on the other hand. Robust skewness is the only measure that on average leads to a risk deterioration of the optimized portfolio compared to the benchmark. Regarding loss-oriented utility, we state a slightly worse performance compared to the remaining risk measures.

We do not want to go into more detail regarding the average improvement and deviation

figures. Since it is undue to compare percentage reductions in risk measures with different growth characteristics. More precisely, loss-oriented utility grows exponentially, whereas semivariance exhibits a quadratic increase, while semideviation grows linearly. Finally, robust skewness is bounded by the interval $[-1, 1]$.

Instead, we look at holding periods, where the objective function of the optimized portfolio exhibits an improvement towards the benchmark. Note that the direction of change does not say much about the accuracy of the predicted risk measures in absolute terms. However, portfolio managers who compare their performance with benchmarks, are more interested in benchmark-relative success anyway. These so called hit ratios are given in the last column of Table 1.

For five out of seven investigated risk measures we find ratios far greater than 50%, indicating a reduction of the respective risk measure in the vast majority of the cases. For significance testing we assume that the number of intended objective changes is a realization of a binomially distributed variable $Y \sim B(Z, p)$ with $p = 0.5$ and $Z = 46$, i.e. the result of a random guess. To reject this hypothesis at the 5% (1%) level, Y needs to be greater or equal to 29 (31). The improvement obtained from the examined optimization procedures is significant for all risk measures under examination except for the skewness related ones. Of the remaining measures, again only loss oriented utility performs little worse than its competitors.

The results of Table 1 suggest that there is considerable persistence in the stock market regarding various risk measures except for skewness-related ones.

To verify this hypothesis we examine the stability of the respective stock rankings, i.e. we determine the ranking of the stock universe with respect to each risk measure and compare the respective orders for all pairs of available consecutive years by means of Spearman's rank correlation coefficient. The results in Table 2 indicate that the risk measures showing the highest correlation coefficient from period to period are volatility, semideviation and semivariance, whereas skewness is not found to be persistent at all. These findings strongly suggest that forecastability of employed risk measures is key for optimization procedures to be practically applicable.

Table 2 about here.

If stock returns do not exhibit strong asymmetry or even come close to symmetry, it is

obvious that optimization with respect to portfolio semivariance is similar to optimization with respect to variance. Moreover, the relationship between variance and semivariance should be stronger in this case.

To examine the interdependence among examined risk measures in our sample, we estimate linear regressions between

- semideviation versus volatility
- square root of semivariance versus volatility
- logarithm of expected (loss oriented) utility versus volatility
- maximum drawdown versus mean and volatility

and test these relationships in a cross-sectional regression at the single stock level. We use four disjoint 3-year periods to estimate the respective historic risk measure values for each of the 40 stocks included. We collect the regression results in Table 3.

Table 3 about here.

Table 3 reveals that semideviation can be virtually completely substituted by volatility in the examined sample. Moreover, the more volatility-related risk measures in general show higher reliability in the above tests. Semideviation and Semivariance perform excellently and show a tight relationship to volatility. Loss-oriented utility and maximum drawdown exhibit significant reliability together with slight deficiencies. For these measures, only about two thirds of their deviation from the mean can be attributed to a linear relationship with volatility.

5 Conclusion

Since investors are risk averse with respect to losses various asymmetric risk measures focusing on downside risk have been proposed in the literature. In this paper we empirically examined the characteristics of asymmetric risk measures such as semideviation, semivariance, loss oriented utility, skewness measures as well as maximum drawdown in an out-of-sample context. Our results indicate that portfolio optimization techniques may successfully reduce asymmetric risk when compared to a strategy of buying and holding a benchmark portfolio. This is in particular true for semideviation, semivariance, loss-oriented utility as well as maximum drawdown while skewness risk is the hardest one to

control. Another important finding is that predictability of alternative risk measures is key for their implementation in portfolio optimization processes. As we applied straightforward estimates derived from historic data for the investigated risk measures, in our case predictability is synonymous with stability over time. Our empirical evidence suggests that skewness measures are highly unstable over time. This observation even holds for a more robust estimator of skewness with regard to outliers. As a consequence, we fail to predict skewness risk properly and optimization fails to reduce skewness risk ex post.

Furthermore, given little asymmetry in equity returns the added value of alternative risk measures compared to the traditional volatility measure might be limited. Note that if asset returns are more or less symmetric we find alternative risk measures such as semi-variance to be closely linked to the traditional volatility measure.

To summarize, alternative risk measures do a good job in portfolio optimization. In order to improve their added value over traditional approaches future research should improve the forecasting techniques for these measures on the one hand and should examine their characteristics in the presence of asset returns facing strong asymmetry.

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Table 1: Risk Results from Various Portfolio Strategies

Strategy	Risk Benchmark	Risk Optimized	Risk Feasible	Risk Reduction (std dev.)	Hit Rate (% of 46)
Volatility	2.538	2.268	1.868	39.27 % (43.55%)	82.6% (*)
Semideviation	0.9726	0.8925	0.6644	25.48 % (28.61%)	82.6% (*)
Semivariance	3.679	2.856	2.094	37.26% (43.65%)	84.8% (*)
Std Skewness	-0.1491	0.0656	-2.490	2.24% (54.88%)	47.8%
Rob. Skewness	0.0656	0.0099	-0.7823	-13.90% (92.21%)	54.3%
Utility	33.43	17.40	7.916	22.90% (62.00%)	76.1% (*)
Max DD	0.07337	0.05381	0.03425	33.76% (43.89%)	82.6% (*)

The first column shows the average values of the respective risk measure obtained from holding the benchmark portfolio. The optimized portfolios reach the values given in the second column, e.g. we yield an average robust skewness of 0,0099 over all holding periods, when we optimize for robust skewness. The values feasible assuming perfect information are shown in the third column, i.e., these are the values for the best portfolios under the given constraints. E.g., the average smallest possible drawdown over all holding periods was 3.425%. The fourth column shows how much of the potential improvement is translated into the optimized portfolios on average. E.g. the gap between benchmark utility and best possible utility is filled by the optimizer by 22,90% on average. The standard deviation of this improvement, expressed in percentage, is given in brackets. Note, that the numbers in column four can not be derived from the first three columns.

The last column gives the percentage of holding periods that show the desired improvement of the respective objective function, i.e. in 35 (or 76,1%) out of 46 observed holding periods the utility-optimized portfolio actually show a higher utility than the benchmark. “(*)” stands for statistical significance.

Table 2: Rank Correlation of Various Risk Measures

years	vola	SD	SV	Skew	rob. Skew	util.	MDD
1993 vs 94	0.7173	0.7578	0.7587	0.4283	-0.0049	0.3655	0.2559
94 vs 95	0.8411	0.8698	0.8392	-0.1248	-0.0553	0.4392	0.2206
95 vs 96	0.6704	0.6569	0.6608	-0.1576	-0.0246	0.4677	0.3775
96 vs 97	0.4310	0.5268	0.4255	-0.0790	-0.0143	-0.0124	0.1668
97 vs 98	0.4732	0.3981	0.3803	-0.0174	0.0568	0.0927	0.1516
98 vs 99	0.3863	0.4150	0.3730	-0.1497	0.0850	0.2670	-0.1383
99 vs 00	0.6516	0.6595	0.6766	0.3143	0.1934	0.0595	-0.0908
00 vs 01	0.3991	0.4629	0.3998	-0.2092	0.1428	0.1413	0.2955
01 vs 02	0.6505	0.7520	0.6565	-0.1756	-0.0013	0.3379	0.5657
02 vs 03	0.7103	0.7587	0.7781	0.2917	-0.0356	0.5296	0.4569
03 vs 04	0.7328	0.7672	0.6882	0.1118	-0.0811	0.3092	0.3683
04 vs 05	0.4402	0.5283	0.3762	0.0218	-0.1964	0.1191	0.1295
mean ρ	0,5920	0,6295	0,5844	0,0212	0,0054	0,2597	0,2300
std. dev. ρ	0,1559	0,1580	0,1789	0,2172	0,1037	0,1768	0,2065

The table reports rank correlation statistics of estimated risk measures from pairs of consecutive years. If we sort the stocks included in this study by their volatility (of weekly returns) observed in 1995 on the one hand, and we sort the same set of stocks by their observed volatility in 1996 on the other hand, the two rankings exhibit a rank correlation of 0.6704. The abbreviated column titles stand for volatility (vola), semideviation (SD), semivariance (SV), standard skewness (Skew), robust skewness (rob. Skew), utility (util.) and maximum drawdown (MDD).

Table 3: OLS Regressions of Risk Measures Against Volatility

Strategy	Equation Strategy vs Volatility	R^2
Semideviation	$sd_i = \begin{matrix} -0.11 & -0.35\sigma_i + \epsilon_i \\ (-4.64) & (-64.85) \end{matrix}$	96,38%
Semivariance	$\sqrt{sv_i} = \begin{matrix} -0.04 & +0.71\sigma_i + \epsilon_i \\ (-1.25) & (92.20) \end{matrix}$	98,18%
Loss Oriented Utility	$\log u_i = \begin{matrix} -1.85 & +1.82\sigma_i + \epsilon_i \\ (-3.78) & (17.08) \end{matrix}$	64,87%
Maximum Drawdown	$md_i = \begin{matrix} -0.07 & +0.11\sigma_i + \epsilon_i \\ (-2.99) & (22.15) \end{matrix}$	64,68%

Cross-sectional regressions of various risk measures versus volatility. We compute estimates for single stock volatility as well as the other risk measures from samples of discrete weekly returns ranging from 1993 to 1995, from 1996 to 1998, from 1999 to 2001 and from 2002 to 2004. The cross-sectional regression is then based on these $40 \cdot 4$ dates. The numbers in brackets give the corresponding t -statistics.

Figure 1:

Slope of exponential utility function

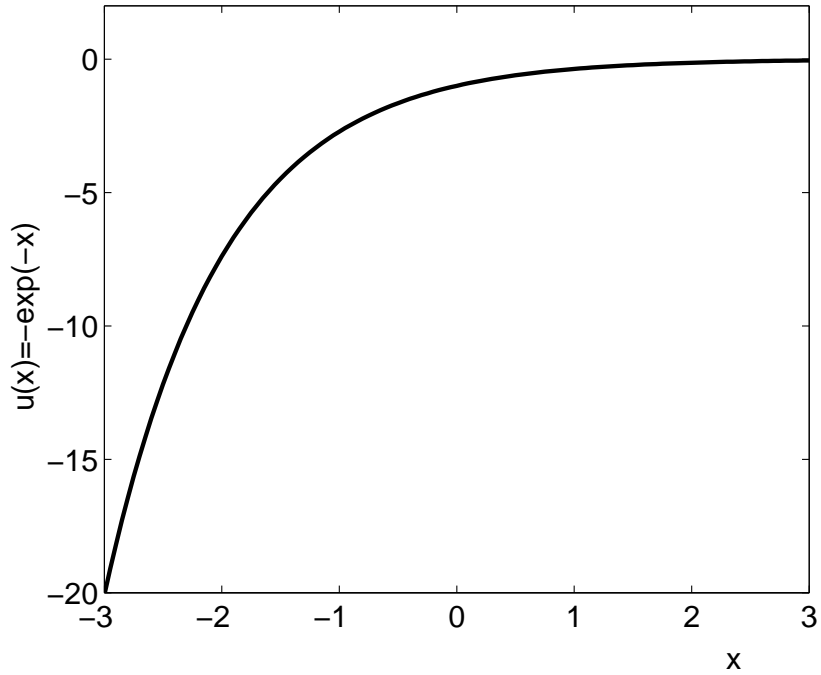
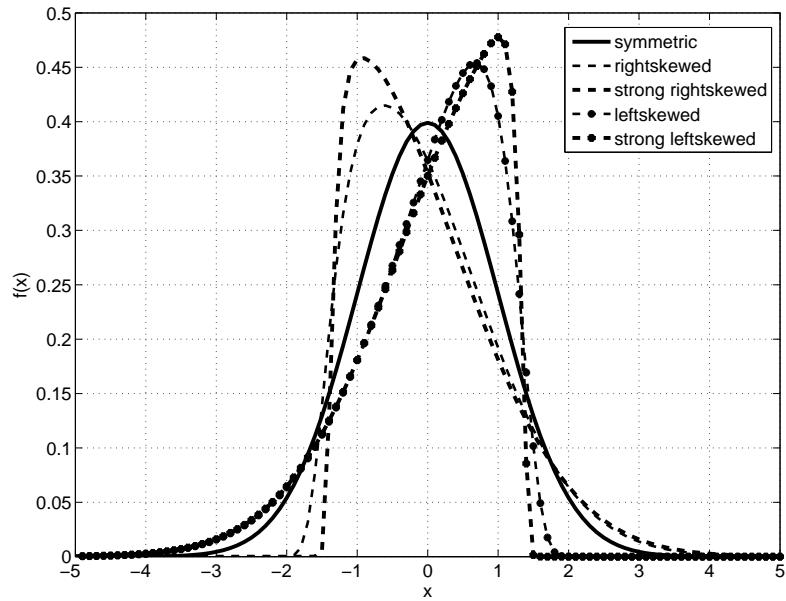


Figure 2:

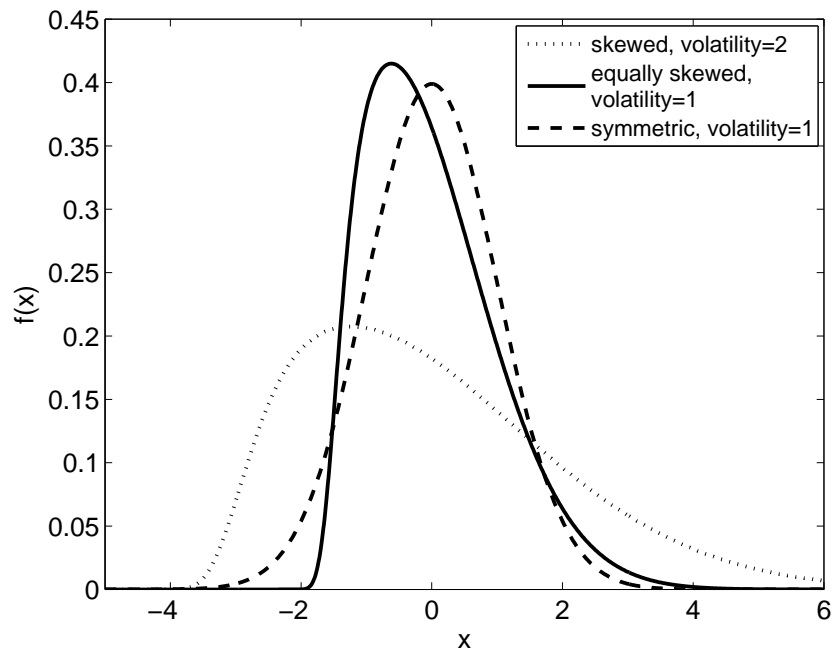
example density functions for skewed distributions



All plotted density functions exhibit the same zero mean and are scaled to a volatility equal to 1. The more positively skewed (right skewed) density functions refer to random variables that show significantly less events at the lower tails. Accordingly, for a downside risk oriented investor the dotted curves correspond to more risky assets.

Figure 3:

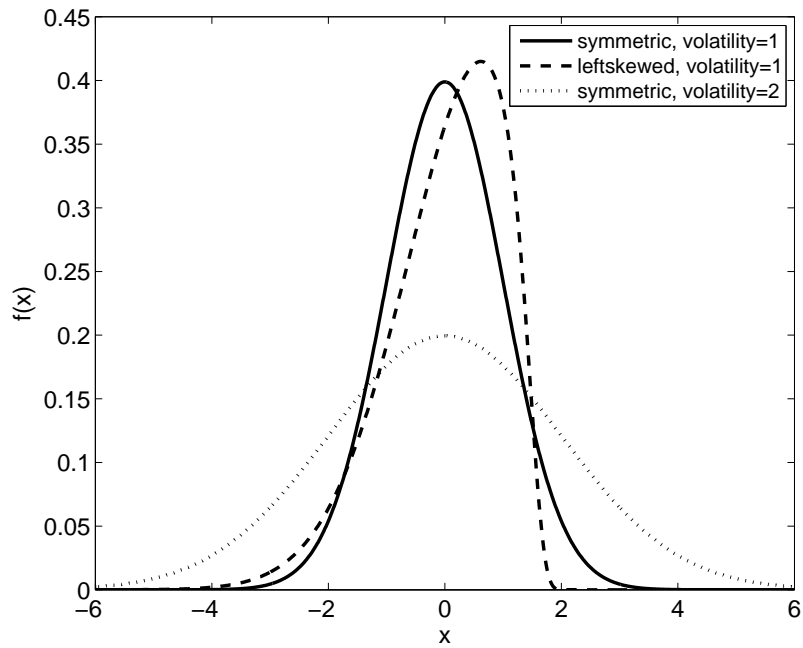
Skewness on its own won't do the trick



Two equally right-skewed distribution functions with equal and double volatility, compared to symmetric distribution. The behavior at the low tails is best for the skewed, low volatility distribution and most adverse for the skewed distribution with higher volatility.

Figure 4:

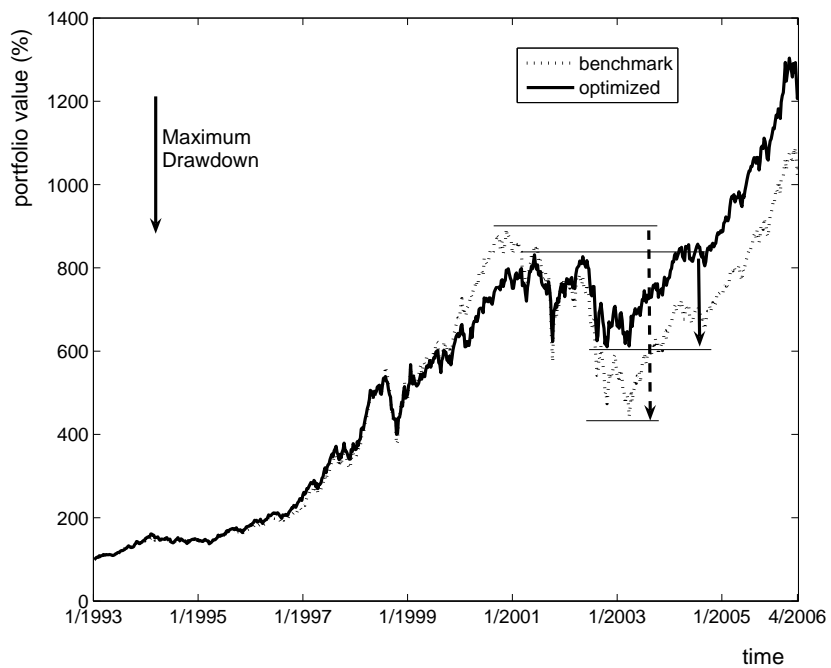
Skewness on its own won't do the trick II



Negatively skewed and symmetric density functions with equal and double volatility.

Figure 5:

Maximum Drawdown of time series



Performance visualization and maximum drawdown: The dashed line gives the results for the benchmark and the solid line gives the performance of a portfolio optimized with respect to maximum drawdown.