Global Derivatives and Risk Management 2004

How To Construct Hedge Fund Portfolios And Structured Products Using A Robust Quantitative Framework

Outline

1. Problem Statement
2. Alternative Sharpe Ratio
3. Aggregating from Positional Information
4. Sample Structured Product
5. Summary

1.1 Problem Statement

Investor interest in hedge funds is likely to grow as interest rates rise. They want:

- Scalability
- Optimal fee structure
- Statistical evidence of adding value
- Fund of Funds (FOF) is a legacy format

1.2 In Theory

Sophisticated investors want to invest with *some* quantitative discipline

- "Lopsided" return profiles of put options
- Uncorrelated "alphas", multiple bets
- Mitigate tail-risk with actuarial approach
- Over-diversification destroys return characteristics

Challenge: A non-trivial statistical problem

1.3 In Practice

Traditional FOFs tend to act like stock pickers or "fund stackers"

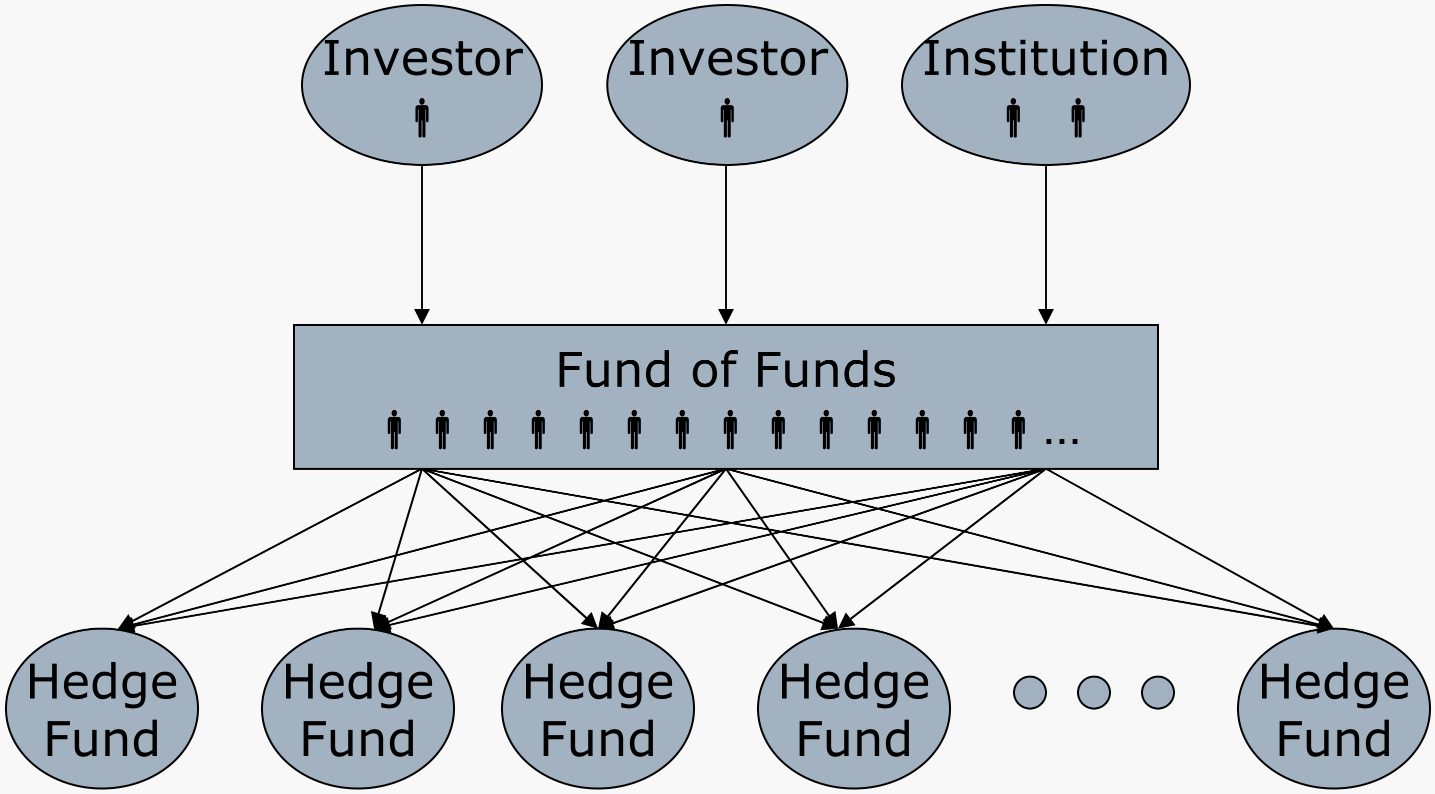
- Capacity and liquidity constraints
- Scale only by increasing headcount
- Minimal use of analytical approach

Opportunity created by analytical approach

- *Lower operational costs*
- *Exciting return profiles*
- *Wider adoption of hedge funds*

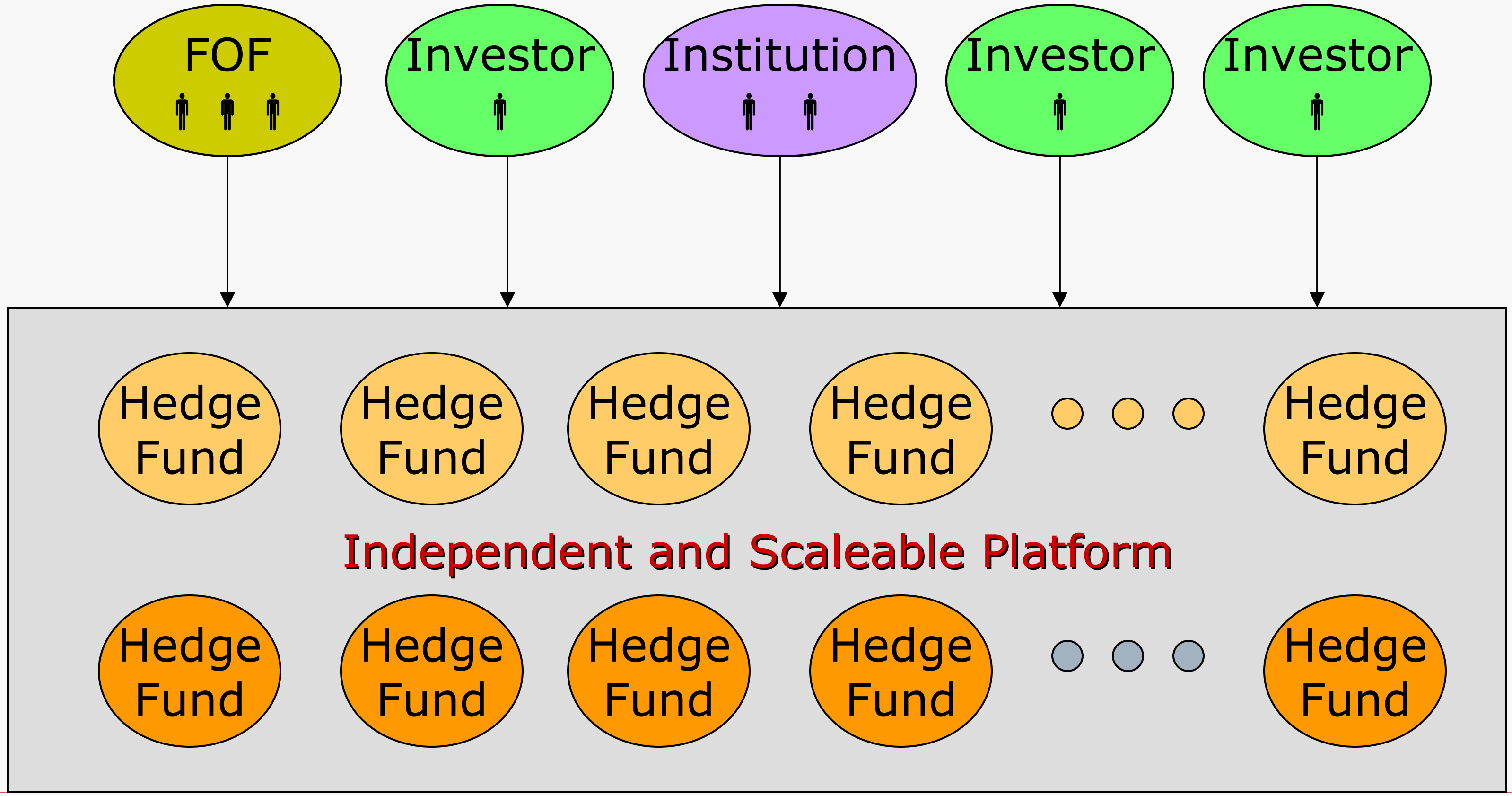
1.4 No-Win Situation

Scaleable Only by Increasing FOF Headcount



1.5 Win-Win Scenario

Investor Selects Scale: Drives Increased Adoption



1.6 Schematic

Investment Advisor

Sales and Distribution

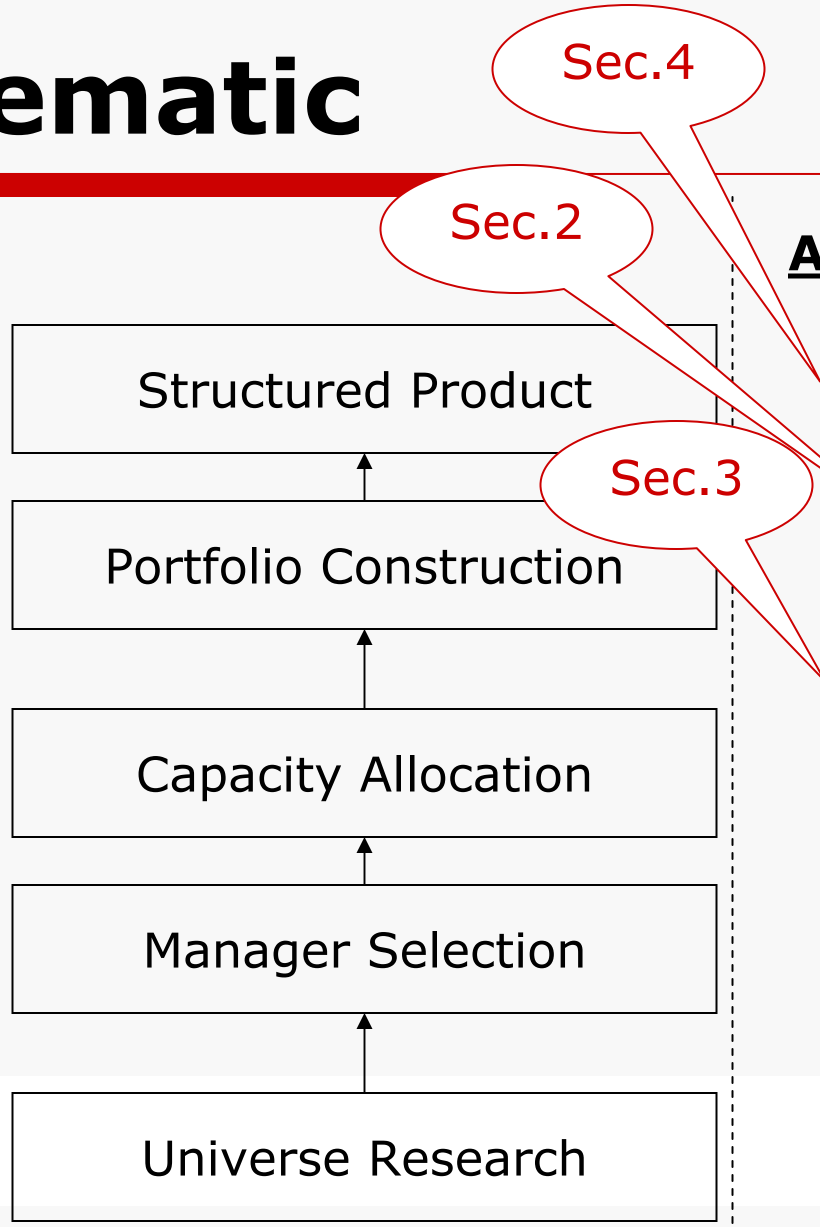
Fundamental Constraints

Investment Outlook

Operational Due Diligence

Relationship Development

Manager Sourcing



Analytics Platform

Structured Product Advisory

Quantitative Portfolio Construction

Manager Exposure Monitoring

Manager Rating

Basic Manager Due Diligence

Data Integration and Quality Assurance

2.1 Risk-Adjusted Drift

- Define the logarithmic portfolio return process as

$$d\log Z_\pi(t) = \gamma_\pi(t)\,dt + \sum_{i=1}^{n} \pi_i(t) \sum_{j=1}^{m} \xi_j(t)\,dW_j(t)$$

- Applying Ito's Lemma to $Z_\pi(t) = \exp\left(\log Z_\pi(t)\right)$

$$dZ_\pi(t) = Z_\pi(t)\,d\log Z_\pi(t) + \frac{1}{2} Z_\pi(t)\,d\left\langle \log Z_\pi(t) \right\rangle_t$$

$$= \left[\gamma_\pi(t) + \frac{1}{2} d\left\langle \log Z_\pi(t) \right\rangle_t\right] Z_\pi(t)\,dt + Z_\pi(t) \sum_{i=1}^{n} \pi_i(t) \sum_{j=1}^{m} \xi_j(t)\,dW_j(t)$$

where $d\left\langle \log Z_\pi(t) \right\rangle_t = \sum_{i=1}^{n}\sum_{j=1}^{n} \pi_i(t)\pi_j(t)\,d\left\langle \log X_i, \log X_j \right\rangle_t = \sum_{i=1}^{n}\sum_{j=1}^{n} \pi_i(t)\pi_j(t)\sigma_{ij}^2(t)\,dt$

- Combining, instantaneous drift rate of the logarithmic portfolio return process $d\log Z_\pi(t)$ given by

$$\gamma_\pi(t) = \sum_{i=1}^{n} \pi_i(t)\gamma_i(t) + \frac{1}{2}\sum_{i=1}^{n} \pi_i(t)\sigma_{ii}^2(t) - \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} \pi_i(t)\pi_j(t)\sigma_{ij}^2(t)$$

2.2 Tail-Risk Adjustment

- Cornish-Fisher Expansion

$$z_{cf} = z_C + \frac{1}{6}\left(z_C^2 - 1\right)S + \frac{1}{24}\left(z_C^3 - 3z_C\right)K - \frac{1}{36}\left(2z_C^3 - 5z_C\right)S^2$$

S – Skewness, K – Kurtosis

- $$\frac{\partial \mathrm{SR}_{cf}}{\partial \pi_1} = \frac{e_1 z_{cf}\sigma - \sum_i \pi_i e_i \frac{\partial z_{cf}\sigma}{\partial \pi_1}}{\left(z_{cf}\sigma\right)^2} = 0$$ where $$\frac{\partial z_{cf}\sigma}{\partial \pi_1} = \sigma \frac{\partial z_{cf}}{\partial \pi_1} + z_{cf}\frac{\partial \sigma}{\partial \pi_1}$$

$$\Leftrightarrow \quad e_1 z_{cf}\sigma = \sum_i \pi_i e_i \frac{\partial z_{cf}\sigma}{\partial \pi_1}$$

$$\Leftrightarrow \quad \frac{e_1}{\frac{\partial z_{cf}\sigma}{\partial \pi_1}} = \frac{\sum_i \pi_i e_i}{z_{cf}\sigma}$$

2.3 Alternative Sharpe Ratio

- "Obvious" definition of the stochastic-term and tail-risk adjusted Sharpe Ratio

$$SR^*_{cf} = \frac{\sum_i e_i \pi_i}{z_{cf}\sigma_\pi} + \frac{1}{2}\frac{\sum_i \pi_i \sigma_i^2}{z_{cf}\sigma_\pi} - \frac{\sigma_\pi}{2z_{cf}}$$

- B. Lee and Y. Lee, "Alternative Sharpe Ratio," in B. Schachter (ed), *Intelligent Hedge Fund Investing*, Risk Books, 2004

$$ASR \equiv \frac{\sum_i e_i \pi_i}{z^-_\pi \sigma_\pi} + \frac{1}{2}\frac{\sum_i \pi_i \left(z_i^+ \sigma_i\right)^2}{z^-_\pi \sigma_\pi} - \frac{1}{2} z^-_\pi \sigma_\pi$$

$$z^+ = \frac{\max\left(z_{cf}\left(z_C^+\right), 0\right)}{z_C^+}$$ where z_C^+ is critical value for probability α and

$$z^- = \frac{\max\left(z_{cf}\left(z_C^-\right), 0\right)}{z_C^-}$$ where z_C^- is critical value for probability $1-\alpha$

(e.g. z_C^+ = 2.33 at 1%, z_C^- = -2.33 at 99%).

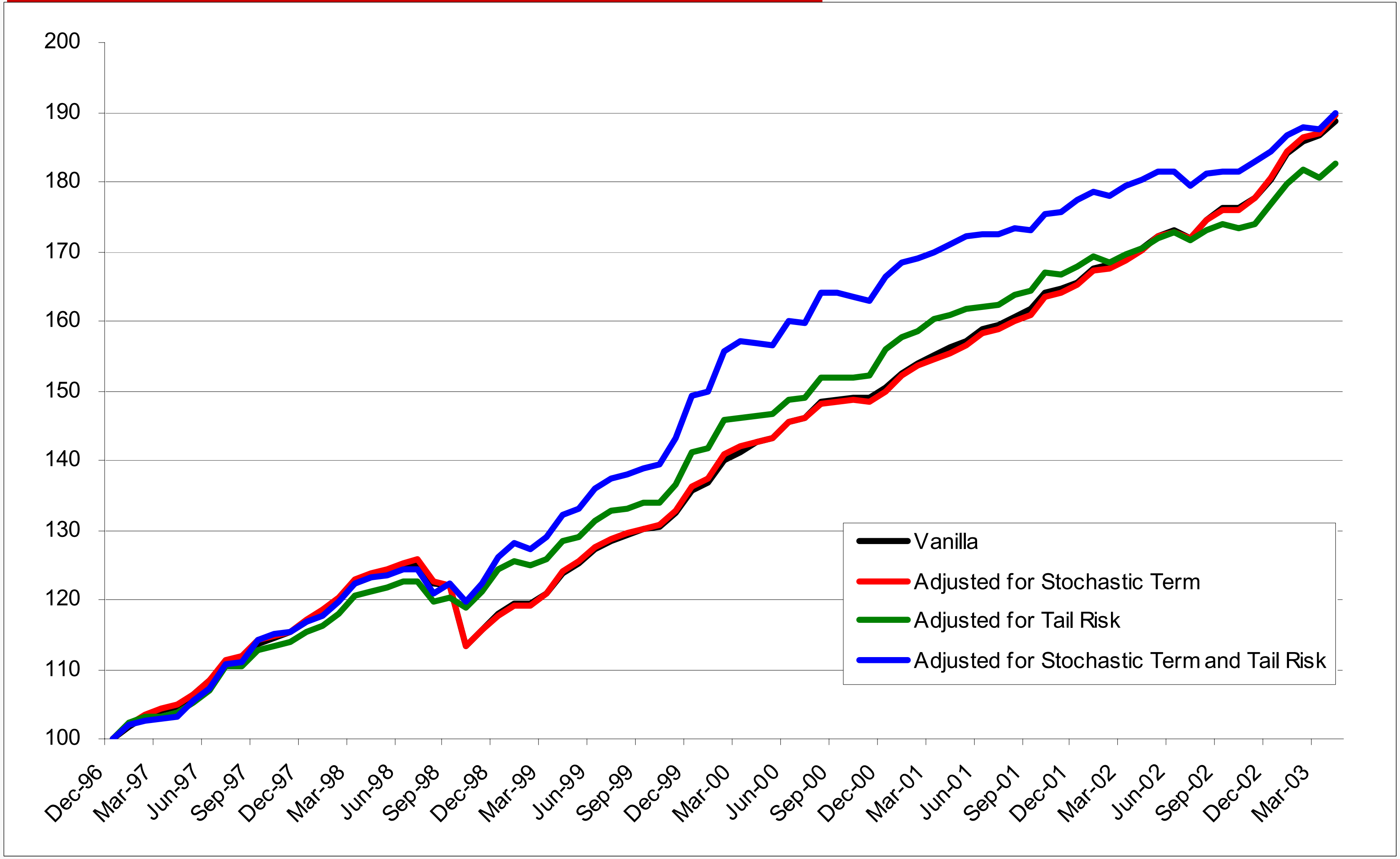
2.4 Axioms

- ASR for single underlying asset with log-normal distribution of returns reverts to traditional SR
- ASR for portfolio of underlying funds with uncorrelated log-normal distributions of returns reverts to traditional SR.
- If underlying assets has log-normal returns, i.e. $z^{+} = z^{-} = 1$ ASR reverts to log-normal case for the analytically correct risk-adjusted Sharpe Ratio based on portfolio gain process proposed by Karatzas and Shreve
- ASR will decrease when $z^{-} > 1$ and/or $z^{+} < 1$ (as in lopsided distributions)

2.5 Upside and Tail Adj.

99% Confidence	**Upside Adj.** z_i^+	**Tail Adj.** z_i^-	**Ratio** z_i^- / z_i^+
HFRI Merger Arbitrage Index	1.2	2.5	2.2
AHFP HFRI Arbitrage Composite (Quarterly Rebalanced)	1.2	2.5	2.0
Fixed Income Proxy Manager	4.9	9.9	2.0
HFRI Distressed Securities Index	1.2	2.2	1.8
HFRI Convertible Arbitrage Index	1.1	1.8	1.7
HFRI Equity Market Neutral Index: Statistical Arbitrage	0.9	1.1	1.2
MAR CTA Index	1.0	1.0	0.9
HFRI Equity Market Neutral Index	1.1	0.9	0.8
HFRI Equity Hedge Index	1.2	1.0	0.8
HFRI Macro Index	1.2	0.9	0.7

2.6 Numerical Example



2.7 Sample Product

Cash substitute for VCs and P&C insurers

- Multistrategy product using HFRX
- Minimal manager idiosyncratic risk
- No capacity concerns in practice
- Weekly liquidity
- Rationale
  - Style allocation accounts for 60-70% of performance
  - Manager selection adds values, but dampens ability to make tactical style bets

3.1 Aggregation from Positions

- Optimality Condition under Alternative Sharpe Ratio

$$MASR = ASR + z_{\pi}^{-}\sigma_{\pi}$$

- "Marginal Alternative Sharpe Ratio" is defined as

$$MASR \equiv \frac{e_i\pi_i + \frac{1}{2}\left(z_i^{+}\sigma_i\right)^2 \pi_i}{\pi_i \frac{\partial z_{\pi}^{-}\sigma_{\pi}}{\partial \pi_i}} = \frac{e_i + \frac{1}{2}\left(z_i^{+}\sigma_i\right)^2}{\frac{\partial z_{\pi}^{-}\sigma_{\pi}}{\partial \pi_i}}$$

where

$$\frac{\partial z_{cf}\sigma}{\partial \pi_1} = \sigma \frac{\partial z_{cf}}{\partial \pi_1} + z_{cf} \frac{\partial \sigma}{\partial \pi_1}$$

3.2 Higher Cross-Moments

- Tail-Risk Contribution $\frac{\partial z_{cf}\sigma}{\partial \pi_1} = \sigma \frac{\partial z_{cf}}{\partial \pi_1} + z_{cf} \frac{\partial \sigma}{\partial \pi_1}$

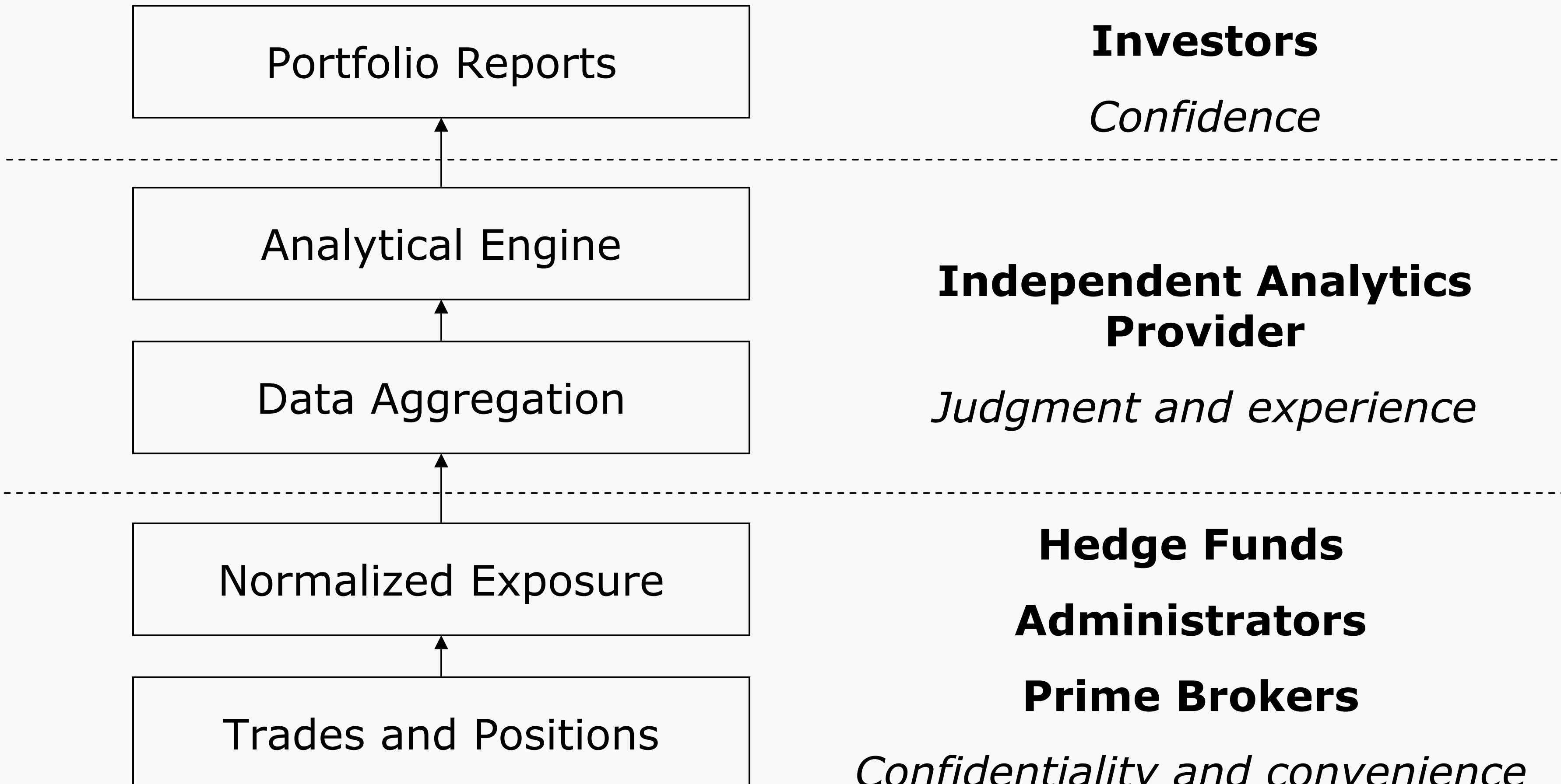
  where

$$\frac{\partial z_{cf}}{\partial \pi_1} = \frac{1}{6}\left(z_C^2 - 1\right)\frac{\partial S}{\partial \pi_1} + \frac{1}{24}\left(z_C^3 - 3z_C\right)\frac{\partial K}{\partial \pi_1} - \frac{2}{36}\left(2z_C^3 - 5z_C\right)S\frac{\partial S}{\partial \pi_1} + \cdots$$

$$\frac{\partial S}{\partial \pi_1} = 3\sum_i \sum_j \pi_i \pi_j E\left\{\left[\frac{R_{i,t} - E\left(R_{i,t}\right)}{\sigma}\right]\left[\frac{R_{j,t} - E\left(R_{j,t}\right)}{\sigma}\right]\left[\frac{R_{1,t} - E\left(R_{1,t}\right)}{\sigma}\right]\right\} - 3\frac{S}{\sigma}\frac{\partial \sigma}{\partial \pi_1}$$

$$\frac{\partial K}{\partial \pi_1} = 4\sum_i \sum_j \sum_k \pi_i \pi_j \pi_k E\left\{\left[\frac{R_{i,t} - E\left(R_{i,t}\right)}{\sigma}\right]\left[\frac{R_{j,t} - E\left(R_{j,t}\right)}{\sigma}\right]\left[\frac{R_{k,t} - E\left(R_{k,t}\right)}{\sigma}\right]\left[\frac{R_{1,t} - E\left(R_{1,t}\right)}{\sigma}\right]\right\} - 4\frac{K+3}{\sigma}\frac{\partial \sigma}{\partial \pi_1}$$

3.3 Transparency Management



4.1 European Put Protection

- Vanilla European OTM put protection on a basket of underlying managers
  - Principal protection too expensive and impractical to execute
  - Easier to short corresponding style index than basket of managers
- Standard framework for pricing quanto options
  - Underlying basket: $S_t = S_0 \exp\left(\sigma_1 W_1(t) + \mu_1 t\right)$
  - Index: $C_t = C_0 \exp\left(\rho\sigma_2 W_1(t) + \sqrt{1-\rho^2}\,\sigma_2 W_2(t) + \mu_2 t\right)$
- Covariance denoted by $\rho\sigma_1\sigma_2$
- Risk-free rate r given by dollar cash bond $B_t = \exp(rt)$

4.2 Replicating Portfolio

- "Replicating portfolio" defined as $\Pi = V(S) - \Delta C$, thus $d\Pi = dV - \Delta dC$
- By Ito's Lemma, $dV = \left(\mu_1 S \frac{\partial V}{\partial S} + \frac{1}{2}\sigma_1^2 S^2 \frac{\partial^2 V}{\partial S^2} + \frac{\partial V}{\partial t} \right) dt + \sigma_1 S \frac{\partial V}{\partial S} dW_1$
- Accordingly,

$$d\Pi = \sigma_1 S \frac{\partial V}{\partial S} dW_1 + \left(\mu_1 S \frac{\partial V}{\partial S} + \frac{1}{2}\sigma_1^2 S^2 \frac{\partial^2 V}{\partial S^2} + \frac{\partial V}{\partial t} \right) dt - \Delta\left(\rho\sigma_2 C dW_1 + \sqrt{1-\rho^2}\sigma_2 C dW_2 + \mu_2 C dt \right)$$

$$= \left(\sigma_1 S \frac{\partial V}{\partial S} - \Delta\rho\sigma_2 C \right) dW_1 - \Delta\sqrt{1-\rho^2}\sigma_2 C dW_2 + \left(\mu_1 S \frac{\partial V}{\partial S} + \frac{1}{2}\sigma_1^2 S^2 \frac{\partial^2 V}{\partial S^2} + \frac{\partial V}{\partial t} - \mu_2 \Delta C \right) dt$$

- If we choose $\Delta = \frac{\sigma_1 S}{\rho\sigma_2 C}\frac{\partial V}{\partial S}$, then

$$d\Pi = \left(\sigma_1 S \frac{\partial V}{\partial S} - \frac{\sigma_1 S}{\rho\sigma_2 C}\frac{\partial V}{\partial S}\rho\sigma_2 C \right) dW_1 - \frac{\sigma_1 S}{\rho\sigma_2 C}\frac{\partial V}{\partial S}\sqrt{1-\rho^2}\sigma_2 C dW_2 + \left(\mu_1 S \frac{\partial V}{\partial S} + \frac{1}{2}\sigma_1^2 S^2 \frac{\partial^2 V}{\partial S^2} + \frac{\partial V}{\partial t} - \mu_2 \frac{\sigma_1 S}{\rho\sigma_2 C}\frac{\partial V}{\partial S} C \right) dt$$

$$= \frac{\sqrt{1-\rho^2}}{\rho}\sigma_1 S \frac{\partial V}{\partial S} dW_2 + \left(\mu_1 S \frac{\partial V}{\partial S} + \frac{1}{2}\sigma_1^2 S^2 \frac{\partial^2 V}{\partial S^2} + \frac{\partial V}{\partial t} - \mu_2 \frac{\sigma_1 S}{\rho\sigma_2}\frac{\partial V}{\partial S} \right) dt$$

4.3 Analytical Approximation

- Establish "baseline" pricing of this call option by solving PDE $\frac{\partial V}{\partial t}+\frac{1}{2}\sigma_1^2 S^2\frac{\partial^2 V}{\partial S^2}+\hat{\mu}S\frac{\partial V}{\partial S}-rV=0$ where $\hat{\mu}=\mu_1-\frac{\sigma_1}{\rho\sigma_2}(\mu_2-r)$
- Solution to a vanilla European call option (Merton)
  $$V_C=Se^{-(r-\hat{\mu})(T-t)}N(d_1)+Ee^{-r(T-t)}N(d_2)$$
  where $d_1=\frac{\log\left(\frac{S}{E}\right)+\left(r+\frac{1}{2}\sigma_1^2\right)(T-t)}{\sigma_1\sqrt{T-t}}$ and $d_2=\frac{\log\left(\frac{S}{E}\right)+\left(r-\frac{1}{2}\sigma_1^2\right)(T-t)}{\sigma_1\sqrt{T-t}}$
- Corresponding European put
  $$V_P=Ee^{-r(T-t)}N(-d_2)-Se^{-(r-\hat{\mu})(T-t)}N(-d_1)$$
- Next step to estimate size of the "tracking error" term
  $$\frac{\sqrt{1-\rho^2}}{\rho}\sigma_1 S\frac{\partial V}{\partial S}dW_2(t)$$

4.4 Numerical Simulation

Correlation between Basket and Index	Put Value		Hedging Error	
	Average	Stdev	Average	Stdev
50% or better	0.81%	3.19%	0.67%	2.85%
55% or better	0.66%	2.81%	0.67%	2.49%
60% or better	0.55%	2.63%	0.68%	2.12%
65% or better	0.55%	2.66%	0.65%	2.10%
70% or better	0.44%	2.11%	0.55%	1.92%
75% or better	0.24%	1.24%	0.50%	1.44%
80% or better	0.19%	0.80%	0.57%	1.39%
85% or better	0.13%	0.49%	0.55%	1.33%
90% or better	0.02%	0.09%	0.13%	0.53%

4.5 Sample Product

OTM put struck at 85% on baskets with 70% or better correlation to style subindices

- 100 bps reasonable fair value estimate for 1-yr put
- Actual product quoted: 85 bps p.a., 3-yr put with style restrictions
- Compare favorably to 50 bps recovery-value protection product
- Tracking error to improve for broker-dealers writing a pool of such products

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Sales and Distribution

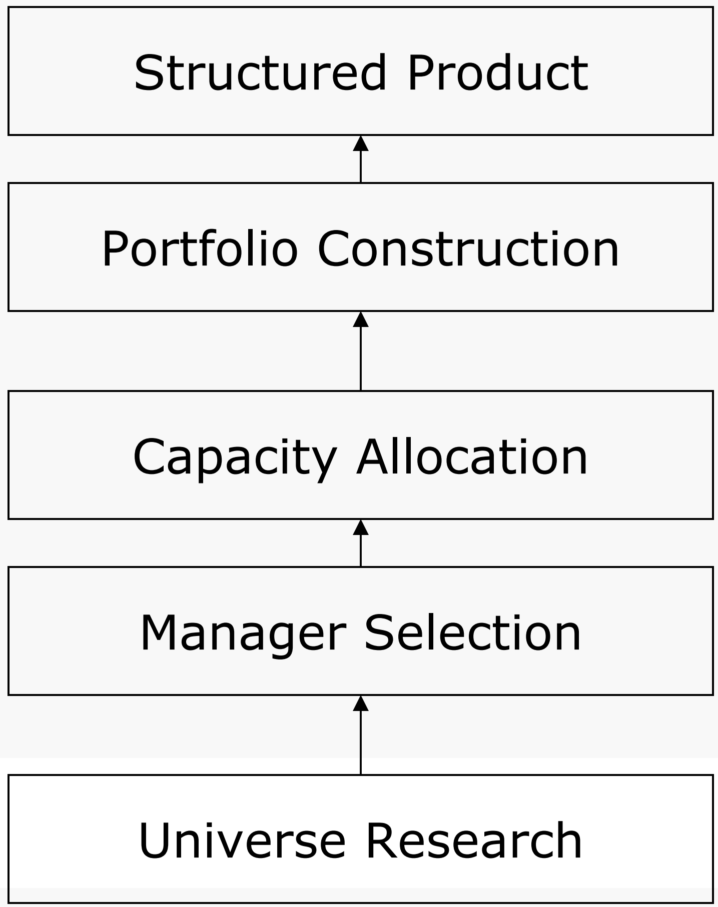
Fundamental Constraints

Investment Outlook

Operational Due Diligence

Relationship Development

Manager Sourcing



Analytics Platform

Structured Product Advisory

Quantitative Portfolio Construction

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Manager Rating

Basic Manager Due Diligence

Data Integration and Quality Assurance

5.2 Final Remarks

Hedge Fund and Structured Product Analytics Platform

- Cost-effective platform to "outsource" quantitative research for sophisticated investors in hedge funds
- Proven analytical approach currently running over $1B
- Team of Princeton, HBS and Stanford alumni based in Cambridge, MA, USA
- Not an investment advisor
- For discussion purposes only

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