Maximum Drawdown Distributions with Volatility Persistence

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Introduction

It is widely recognized that hedge funds and managed futures have unique characteristics that pose challenges to risk management. In particular, the active nature of their investment process can result in returns with non-constant volatility and non-normal return distributions. Because the return distributions may not be symmetrical, hedge fund investors and risk managers are concerned with down-side risk. Drawdown statistics are also a practical measure of liquidity risk, consistent with the nature of investments that are marking to market and using stop orders to control risk.

We define drawdowns as the maximum percentage loss occurring between two high-water marks. A high water mark is said to occur when the net asset value is higher than values preceding and following it. In this chapter, we analyze distributions of *maximum* drawdowns with historical fund returns and Monte Carlo simulations. My results replicate those of previous studies, showing that the maximum drawdown will be larger with a lower mean, higher standard deviation, and longer track record.

The focus of this study however is on the impact of volatility persistence, which provides a partial explanation for the observation of kurtosis (fat tails) in asset returns. Previous studies have shown that kurtosis in fund returns does not impact the distribution of maximum drawdowns. We present evidence that seems to indicate that kurtosis does impact maximum drawdown distributions, but recognize that persistence in volatility may provide a partial explanation. In particular, it is interesting to note that kurtosis influences maximum drawdowns for individual funds, but not for funds of funds. Funds of funds have an average kurtosis similar to individual funds, but are less likely to exhibit volatility persistence. We also present some evidence on which strategies are more likely to exhibit persistence in volatility than others.

In the following sections, we review related literature, describe the data and methodology and present results. The conclusion summarizes the results and discusses avenues of research for furthering our understanding of the use of drawdowns in risk management for alternative investment vehicles.

Previous Literature

There exists an extensive literature on the subject of drawdowns, ranging from the anecdotal to the rigorous, from the practical to the arcane. An important contribution in assessing the distribution of drawdowns is made by Burghardt, Duncan and Liu (2003). In this work, the authors (BDL) explore variables that are potential determinants of drawdowns distribution. In particular, they find that length of track record, mean return and the size of the volatility of a manager's returns account for differences in drawdown distributions. On the other **HAND** they find that skewness and kurtosis in a return series do not affect the distribution. The expected size of a drawdown correlates negatively with the mean return, and positively with volatility. In addition, the authors observe that managers may de-leverage when in a downturn, lowering their volatility. This suggests that the volatility in a manager's return might be changing over time. They pose as questions for further research how changes in a manager's volatility would affect the distribution of drawdowns and maximum drawdowns, and how accounting for serial correlation in a series might affect those distributions. In this work, we tackle both issues by exploring the distribution of drawdowns and maximum drawdowns within a GARCH framework, which accounts for both a time-varying volatility and serial correlation in returns.

In a seminal contribution on the subject, Lopez de Prado and Peijan (2004) explore the effects of non-normality as well as serial correlation assumptions in the return series to measures of risk based on drawdowns. They find via a simulation study that risk is usually under-estimated (in the form of VaR, or percentile estimates of drawdowns) when the distribution of returns is not normal (in particular, they assume that returns follow a mixture of normal distributions). Furthermore, if returns are assumed to follow an ARIMA(p,1,q) process, measures such as VaR does not fully capture all dimensions of risk in the market, and they suggest using measures based on drawdowns instead. While these authors explore both non-Gaussianity and time dependence, they suggest that hedge fund returns exhibit, in addition, conditional volatility regimes. They claim that these different volatility regimes can be captured via their mixture model. In the present work, in contrast, we explore directly the possibility that hedge fund return's serial dependency and departure from Gaussianity might be appropriately modeled by GARCH, and investigate under this paradigm, the distribution of drawdowns and maximum drawdowns.

On a more technical note, de Melo and Camara Leal (unpublished) consider the distribution of maximum drawdowns under the assumption of an integrated GARCH model process for the returns series. They choose a parametric model for the distribution of maximum drawdowns based on extreme value theory. In particular, they assume that the distribution of maximum drawdowns follow a modified generalized Pareto distribution, and using simulated data from integrated GARCH, estimate parameters for this distribution using maximum likelihood. Analyzing data from Nasdaq and the British FTSE, they note that extreme maximum drawdowns tend to occur during periods of high volatility; however, they also note that uncharacteristic drawdowns are also possible in periods of relatively low volatility. In the present work, while we consider the behavior of maximum drawdowns under a GARCH model for the returns, we do not assume a parametric form for the distribution of maximum drawdowns.

Data and Methodology

We use fund return series from the CISDM database that have track records with a minimum of two years and starting in January 1985 or later. The data series end in July 2004. These criteria yield a total of 2071 individual funds and include several distinct hedge fund strategies, commodity trading advisors (CTAs) and funds of funds (FOFs). The summary statistics are presented in Table 1.

<<Insert Table 1 about here>>

The maximum drawdowns, track records and summary statistics are calculated for each fund. The entire sample is divided into two groups based on a dummy variable indicating the presence of volatility persistence based on the Ljung-Box-Pierce Q-test for a departure from randomness based on the autocorrelation functions of the squared residuals from the mean. This is a common pre-estimation step for determining if auto-regressive conditional heteroscedasticity (ARCH) effects **EXIST** in the data, and provides results that are identical to Engle's ARCH test. We test several lags at five and ten percent significance levels. We find that 21.6% of the funds in the sample have volatility persistence in their monthly returns at the five percent significance level, and volatility persistence is indicated in 30.7% of the funds at the ten percent significance level. In the results presented here, we work with the larger sample. Although the results are similar, inferences made from working with the larger sub-sample of funds is actually more conservative than working with the smaller group at a more stringent LBQ test significance level because we are interested in comparing the two groups.

We graph the frequency distributions of the maximum drawdowns for various groups of funds. Samples are formed based on length of track record, presence of volatility persistence, and fund strategy classification. We also run simple regressions with maximum drawdown as the dependent volatility and confirm previous findings relating to mean and standard deviation, and present new evidence on the impact of kurtosis on **DRAWDOWNS AND** maximum drawdowns. In addition, we also calculate the relative frequency of indications of volatility persistence among various fund strategies. Monte Carlo simulation results can be viewed as representing the theoretical maximum drawdown distribution for a single hedge fund obtained from multiple return histories calibrated with a mean and unconditional volatility representative of monthly hedge funds. These results are used to confirm the inferences drawn from the historical returns. This evidence, combined with the regression results on fund of funds, managed futures and hedge fund strategies allow us to draw conclusions about the related impacts of kurtosis and volatility persistence on drawdown distributions.

Results

Figure 1 illustrates the distribution of maximum drawdowns for all funds that do not exhibit volatility persistence as measured by the LBQ statistic. This group of funds is divided into three groups based on the length of the track record. Consistent with BDL (2003), the graph illustrates the result that funds with a longer track record have a larger likelihood of experiencing larger drawdowns. Figure 2 shows that this intuitive result also holds for the group of funds that exhibit return volatility persistence.

<<Insert Figure 1 about here>>
<<Insert Figure 2 about here>>

Figure 3 indicates that funds that exhibit volatility persistence have a higher likelihood of experiencing a greater maximum drawdown. We find this interesting because pervious studies have indicated that kurtosis does not impact the distribution of theoretical maximum drawdowns, and volatility persistence is associated with fat tails. We are also not aware of any other study that illustrates this empirical result. Our following analysis focuses on investigating causes for this result.

<<Insert Figure 3 about here>>

In Table 2 we examine the relative frequency of funds that exhibit volatility persistence in various strategy classifications. The average frequency is 30.7%. For classifications that contain more than 30.7% of the funds that exhibit volatility persistence include currency and financial CTAs and almost all of the hedge fund strategies, particularly short sellers. Hedge funds that do not exhibit a lot of volatility persistence include market neutral and event driven strategies. Volatility persistence **DELETE IS** also occurs with less frequency than average in fund of funds, diversified CTAs and guaranteed products. One difficulty in this analysis is that while no fund is double-counted, the categories are sometimes overlapping. For example, offshore funds may also be a currency CTA but this classification system does not make this distinction. Nevertheless, we are able to draw the broad conclusion that volatility persistence seems more prevalent in less diversified funds.

<<Insert Table 2 about here>>

In Figure 4, we graph simulation results to confirm inference from the historical returns that volatility persistence impacts the distribution of maximum drawdowns. Two sets of 1000 monthly returns representing an 8 year track record are simulated, each with a mean and unconditional volatility approximating the average of the entire sample (See Table 1). The first set represents the case of no volatility persistence: $r_t = \mu_t + \sigma_t \varepsilon_t$, where $\sigma = .04$ and $\mu = .01$. The second set is simulated based on a return generating model where the variance is described by a GARCH (1,1) process:

$$\sigma^2 = \kappa + \sum_{i=1}^{P} G_i \sigma_{t-i}^2 + \sum_{j=1}^{Q} A_j \varepsilon_{t-j}^2$$

with i, j = 1, G = .4, A = .5, and k =**????????** , and u in the return generating process also equal to 0.01. The mean and standard deviations for the first and second set of simulated returns are, respectively. The excess kurtosis for the first set is close to zero at _**????????**, while the kurtosis for the second set is a much higher **??????**. The simulation results clearly illustrate that the returns with volatility persistence have a larger likelihood of a larger maximum drawdowns, confirming our empirical results. Because excess kurtosis is associated with the model's volatility persistence, we return to the historical returns to further investigate the relative impacts on maximum drawdowns.

In Table 3 we group the entire sample into three subsets: 454 fund of funds, 501 managed futures, and 1116 hedge fund strategies. On average, each group exhibits kurtosis of 3.95, 2.35, and 4.05, respectively. OLS regression analyses are performed for each of the three groups where maximum drawdowns are the dependent variable. For all three analyses, the independent variables discussed in BDL (2003) are significant: mean, standard deviation, and length of track record. Also consistent with BDL's results, kurtosis and a dummy variable for volatility persistence **ARE** insignificant in the fund of funds results. However, both kurtosis and the dummy variable are significant for the second two groups.

Conclusion

Our results show that under non-constant volatility, length-track record does impact both the distribution of drawdowns and the distribution of maximum drawdowns. In particular, we observed that for larger length-track records the distribution of the drawdowns presents fat tails, which provides a partial explanation to the difference in the distribution of their maximum drawdowns. This result addresses one of the main questions in the paper by (BDL). It would be interesting to further study analytically what the distribution of the drawdowns and maximum drawdowns might be assuming that the volatility underlying the returns follows a specific time-varying process.

Bibliography

Vaz de Melo, Beatriz and Ricardo Pereira Camara Leal, "Maximum drawdown: Models and Applications". (Http??, Affiliation? Date?)

Check for inclusion of all references mentioned so far?

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Table 1. Overall Summary Statistics

Monthly Data	Track Record	Average Return	Standard Deviation	Kurtosis
MIN	24	-2.302%	0.0063%	-1.2578
MAX	235	6.224%	28.66%	83.983
AVERAGE	83.8	1.0247%	3.998%	3.6182

Figure 1. Frequency of Maximum Drawdowns – Track Records with No Persistence

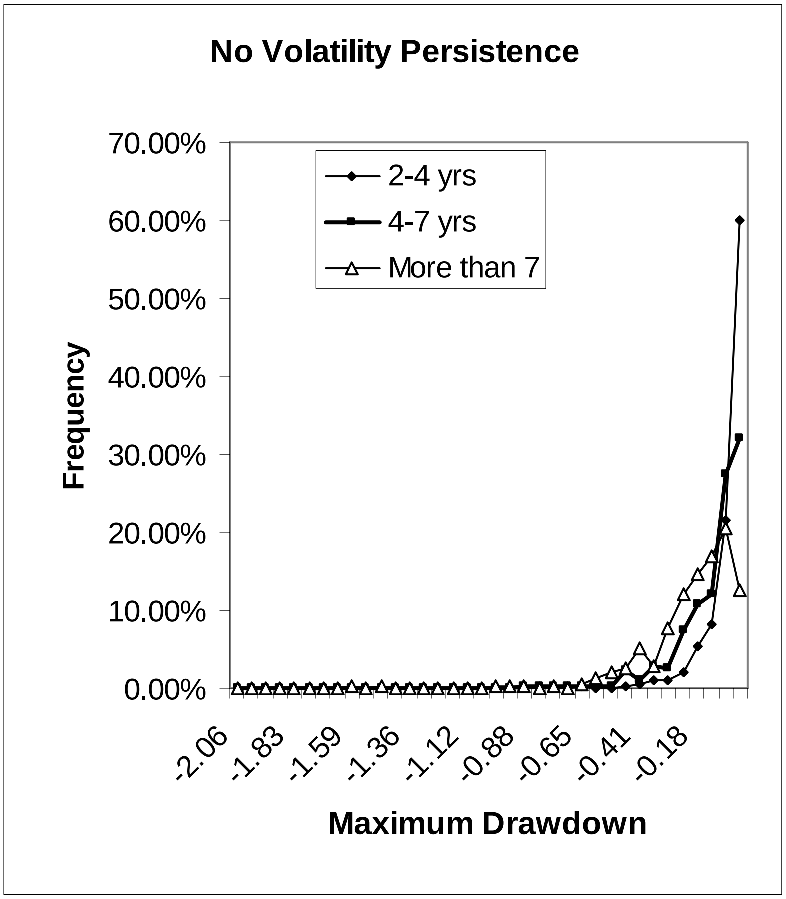

Figure 2. Frequency of Maximum Drawdowns – Track Records with Volatility Persistence

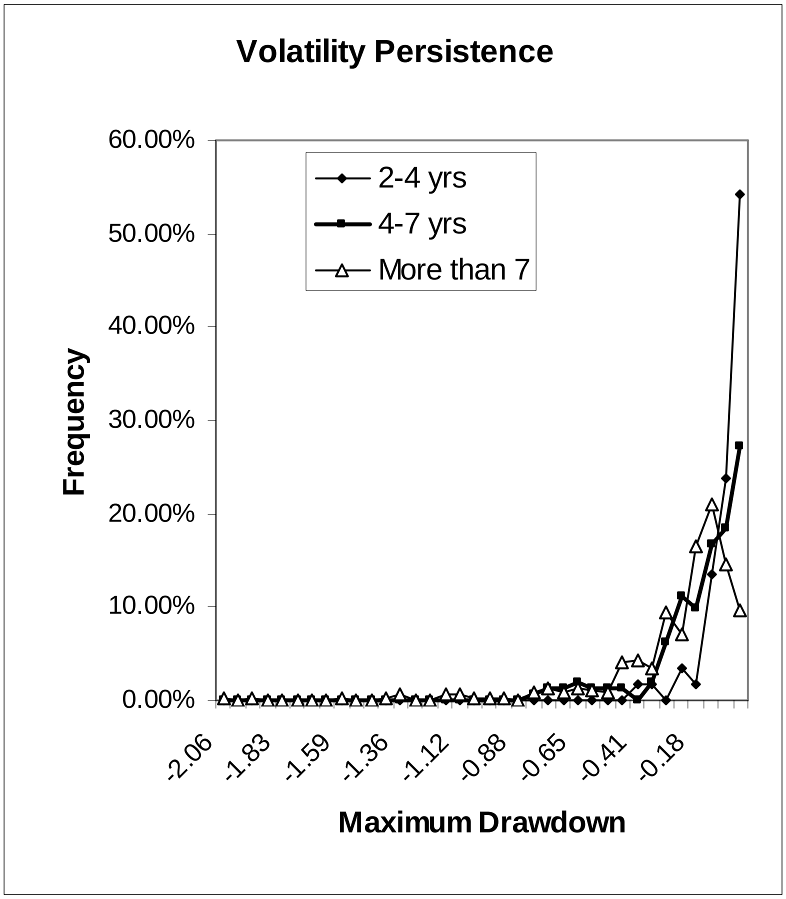

Figure 3. Frequency of Maximum Drawdowns – Volatility Persistence & No Persistence

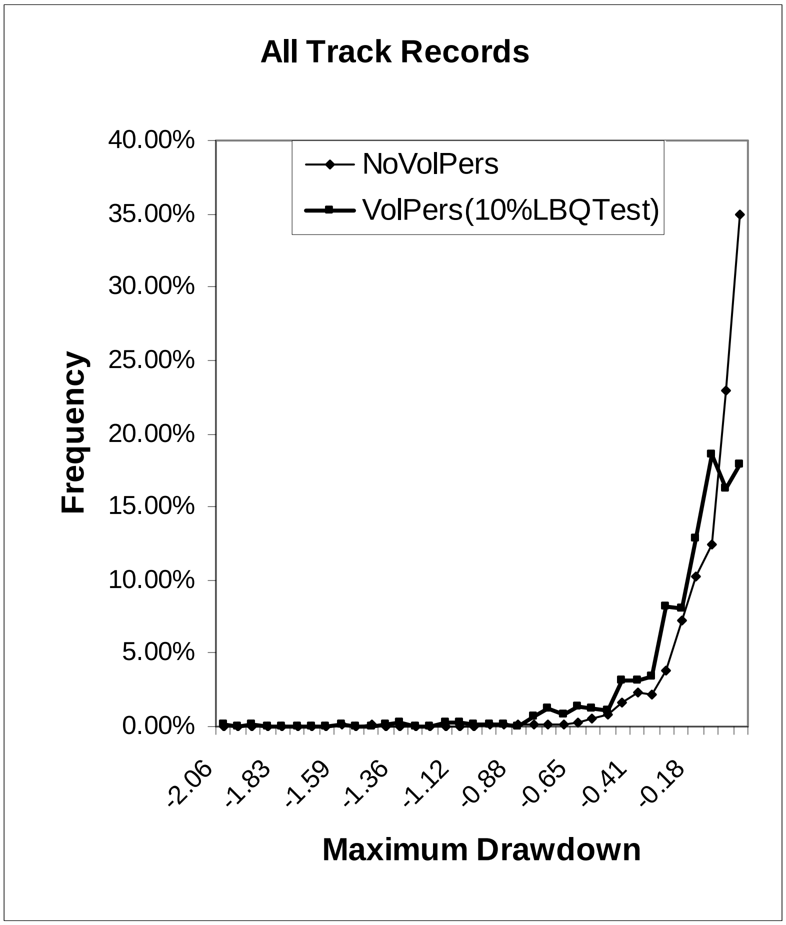

Table 2. Frequency of Volatility Persistence by Hedge Fund Strategy

	w/VolPers	Total	
FOF	132	454	
FUT-CTA-AG/EN/MET	2	15	
FUT-CTA-CUR	17	33	**
FUT-CTA-DIV	40	138	
FUT-CTA-FIN-STX-INDEX	25	68	*
FUT-PUB-GUAR US&NON	2	7	
FUT-PUB-OFFSHORE	24	104	
FUT-PUB-US CLOSED&OPEN	20	46	*
FUT-PVT-US CLOSED&OPEN	32	90	*
HF -NON&US-EVENT DRIV	37	153	
HF -NON&US-GLOBAL MACRO	15	45	*
HF -NON&US-GLOBAL EMER	32	101	*
HF -NON&US-GLOBAL EST	95	281	*
HF -NON&US-GLOBAL INTL	16	35	**
HF -NON&US-LONG ONLY	3	8	*
HF -NON&US-MKT NEUTRAL	99	370	
HF -NON&US-SECTOR	32	107	
HF -NON&US-SHORT-SALES	13	16	***
TOTAL:	636 (30.71%)	2071	

*More likely to exhibit persistence in volatility of monthly returns,
At a 5% level, *, at a 1% level

Figure 4. Simulation Results

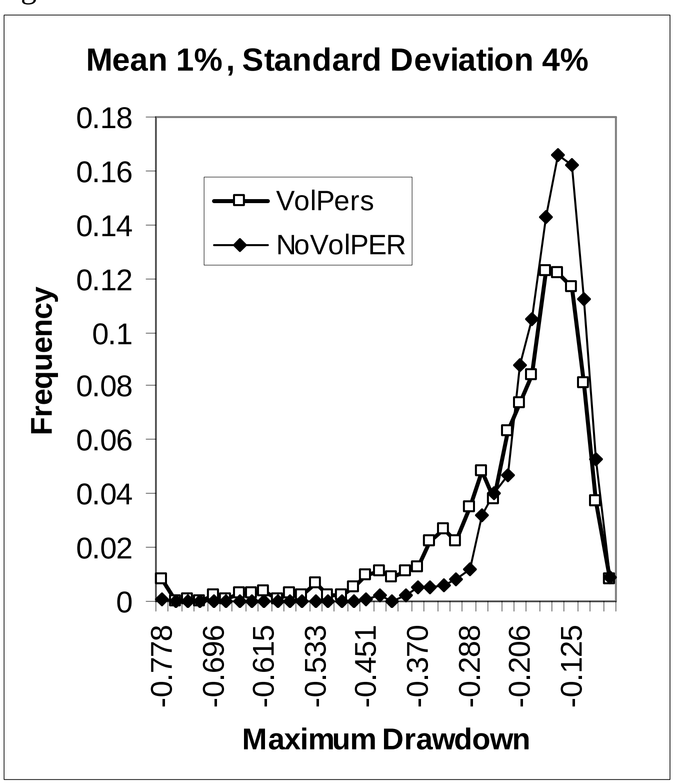

Table 3. Determinants of Maximum Drawdowns: Mean, Volatility, Kurtosis, Track Record, & Volatility Persistence

Fund of Funds, count = 454

	MaxDrawdown	*Ave Return*	*STDEV*	*KURT*	*TrackRecord*	*VPdummy*
Mean	-0.0898	0.0071	0.0188	3.9538	78.5815	0.2907
Coefficients		8.4094	-7.0320	-0.0004	-0.0005	0.0034
t Stat		9.7164	-30.1542	-0.9163	-6.8928	0.5134
P-value		0.0000	0.0000	0.3600	0.0000	0.6079

Futures/CTAs, count = 501

	MaxDrawdown	*Ave Return*	*STDEV*	*KURT*	*TrackRecord*	*VPdummy*
Mean	-0.2171	0.0107	0.0568	2.3540	105.8104	0.3234
Coefficients		6.4727	-5.1107	-0.0029	-0.0006	-0.0177
t Stat		9.9740	-34.7030	-3.5164	-7.4721	-2.1452
P-value		0.0000	0.0000	0.0005	0.0000	0.0324

Hedge Funds, count = 1116

	MaxDrawdown	*Ave Return*	*STDEV*	*KURT*	*TrackRecord*	*VPdummy*
Mean	-0.1847	0.0113	0.0411	4.0492	76.0197	0.3065
Coefficients		5.0935	-5.8702	-0.0014	-0.0008	-0.0205
t Stat		11.1920	-55.7695	-3.4337	-9.6741	-2.9800
P-value		0.0000	0.0000	0.0006	0.0000	0.0029